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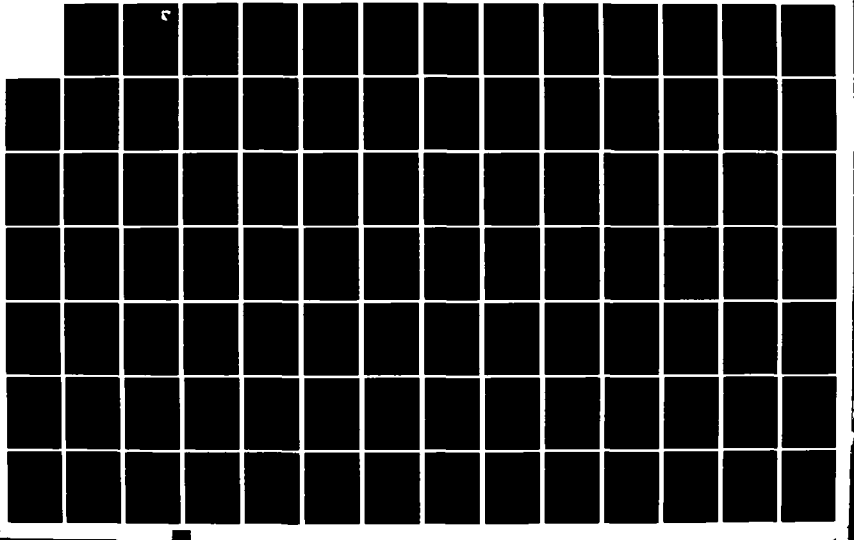

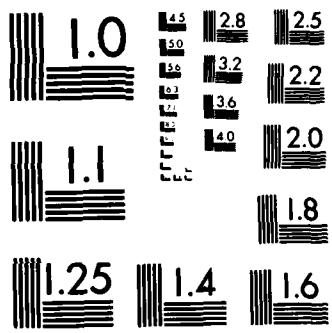

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CONJOINT MEASUREMENT AND CONJOINT SCALING: A USERS GUIDE

APRIL 1982

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This report has been reviewed by the Office of Public Affairs (PA) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

CHARLES BATES, JR.
Chief
Human Engineering Division
Air Force Aerospace Medical Research Laboratory

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Conjoint measurement methodology offers a new and potentially useful approach for obtaining psychological scale values for components of multidimensional attributes. This report describes the mathematical foundations of this methodology. Six computer based algorithms that can be used to perform specific kinds of conjoint analysis have been generalized and documented for application as subjective assessment techniques. The six programs (CONJOINT, PCJM2, NONMETRG, MONANOVA, DISTRIB, and DUALDIST) are each summarized with respect to their function as conjoint analysis techniques. Additionally,

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Block 20. Abstract (continued)

the appendix provides a step-by-step explanation of data deck arrangements for the programs described.

PREFACE

This report describes the foundations of conjoint measurement and the ordinal properties associated with the more widely used polynomial conjoint measurement models. The principles behind several widely used conjoint testing and conjoint scaling computer programs are explained. Additionally, Appendix 1 provides a detailed explanation of how to set up data decks for using the six programs described. This report was one of the products produced by Dr. Thomas E. Nygren while participating in the 1981 USAF-SCEEE Summer Faculty Research. The work was performed in support of AFSC Project 7184, Man-Machine Integration Technology for the Air Force, Air Force Aerospace Medical Research Laboratory (AFAMRL), Human Engineering Division (HE), Wright-Patterson AFB, Ohio 45433.

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I. INTRODUCTION

Subjective scaling techniques are an integral part of much of social science research. In many situations it is assumed that the variable of interest is a complex phenomenon that is multidimensional in nature. That is, it is recognized that the ordering of scores produced by an individual on this variable may be based on the joint effects of two or more independent variables. The multidimensionality of the phenomenon, of itself, poses no real problem to the research since in many cases, standard statistical procedures like analysis of variance or multiple regression techniques may be used. These procedures, however, are primarily used to assess the predictive ability of the independent variables rather than to estimate psychological scale values.

Often the researcher may be interested in one or both of the following more basic issues. First, can the composition rule by which the independent variables combine to produce the joint effect on the dependent variable be established empirically? In addition, it may not be possible to obtain initial measurements for the independent variables themselves, but only for their resultant joint effects. Secondly, then, can the independent and dependent variables be scaled simultaneously according to some specified composition rule in a way that preserves the order of the joint effects in the data? This question, as Tversky (1967) points out, is the conjoint measurement problem, and the composition rule is the conjoint measurement model.

There are, of course, many composition rules that might be hypothesized in psychological theories. The simplest such rule is an additive one which suggests that the independent variables combine in an independent additive fashion to produce the joint effect. For example, let a_1 be a level of factor A_1, a_2 be a level of factor A_2, and a_3 be a level of factor A_3. We might hypothesize that the joint effects of these three factors could be described as

$$f(a_1, a_2, a_3) = f_1(a_1) + f_2(a_2) + f_3(a_3) \qquad (1)$$

where f, f_1, f_2, and f_3 are separate and identifiable numerical functions. Additive models like the three-factor model illustrated in Equation 1 have been and continue to be an important part of many psychological theories. Until recently, however, even for this simple model, there has not been a satisfactory means by which one could simultaneously estimate all four functions f_1, f_2, f_3, and f. Conjoint measurement theory provides a means to do this and herein lies its power. Just as important, however, is the result of the theory (to be described below) which indicates that only ordinal relations are required among the data points in order to produce resultant scales unique up to an affine transformation. The implications of this result will become more apparent following the presentation of the basic theory of conjoint measurement.

II. FOUNDATIONS OF CONJOINT MEASUREMENT

Prior to an introduction to the mathematical foundations of conjoint measurement it might be useful to review two terms that are generally distinguished in the literature (Emery and Barron, 1979; Green and Rao, 1971; Green and Srinivasan, 1978). First, we define conjoint measurement as the procedure whereby we specify for a given combination rule, the conditions under which there exist measurement scales for the dependent and independent variables, such that the order of the joint effects of the independent variables in the data are preserved by the numerical composition rule. We then define conjoint analysis (sometimes referred to as numerical conjoint measurement) as the procedure whereby the actual numerical scale values for the joint effects and the levels of the independent variables are obtained. Thus, there are effectively two separate and independent processes in the conjoint measurement methodology. First, one attempts to find the appropriate combination rule and then, assuming the rule is valid, finds numerical functions that "best" fit the observed order of the joint effects in the data according to the specified rule.

We begin the discussion of the foundations of conjoint measurement with the definition of a decomposable structure.

Definition 1. Given:
(1) the set $A = A_1 \times A_2 \times \ldots \times A_n$ where each of the A_i are non-empty sets, (2) a binary relation $\gtrsim$ on $A = \prod_{i=1}^{n} A_i$ (where "$\gtrsim$" might be a relation like "is prefered to", "is larger than", "is greater in workload than", etc.), and (3) a real-valued function F in n real variables,

then we say that $\langle A_1, A_2, \ldots A_n \rangle$ is decomposable if we can construct real-valued functions ϕ_i on A and ϕ_i' on the A_i', such that for $a = (a_1, a_2, \ldots a_n)$ and $b = (b_1, b_2, \ldots, b_n)$,

(i) $a \gtrsim b$ if and only if $\phi(a) \geq \phi(b)$

(ii) $\phi(a) = F[\phi_1(a_1), \phi_2(a_2), \ldots, \phi_n(a_n)]$

where F gives the composition rule for the n factors. The measurement question at this point is whether we can find the set of axioms or empirical laws that are necessary and/or sufficient to ensure decomposability for the specific function F. If F is an additive composition rule, then we have a specific decomposable structure, an additive conjoint structure.

Decomposability, while certainly necessary for additive conjoint measurement, is not sufficient. To obtain a set of sufficient conditions we must rely on a theorem first proved by Hölder (1901). Thus, the foundations of conjoint measurement have been established for some time, although it was not until Luce and Tukey (1964), Krantz (1964), and Tversky (1967) established conditions for additive and polynomial conjoint measurement, that interest in the measurement technique began to increase.

Luce and Tukey (1964) restricted their initial discussion to establishing the sufficient conditions for additive conjoint measurement in two factors. Krantz (1964) later added some extensions to their work as did Tversky (1967) in his presentation of a general theory of polynomial conjoint measurement. In the Luce-Tukey axiomatization we begin by defining $A = A_1 \times A_2$ to be a set of objects that can be constructed from independent components

of the non-empty sets A_1 and A_2. We again define $\succsim$ as a binary relation on A. We can now present the set of axioms.

Axiom 1. Let $\succsim$ be a weak order on A such that $\succsim$ satisfies the following two properties:

(i) either $a \succsim b$ or $b \succsim a$ and
(ii) if $a \succsim b$ and $b \succsim c$, then $a \succsim c$.

These two properties make $\succsim$ a meaningful empirical relation. Property (i) above insures that all elements in A can be compared with respect to $\succsim$. This is sometimes referred to as the property of connectedness. In addition, a weak order must exhibit property (ii) above, transitivity. Although transitivity seems at first glance to be a very innocuous and trivial axiom, it is not. There are many examples in the psychological literature that show subjects consistently violating transitivity, (cf. Tversky, 1969).

We now come to our first "technical" axiom. It is an axiom that states that we can solve inequalities in our system. Hence, the axiom in its many forms is referred to as solvability. Formally, we have

Axiom 2. Let $a = (a_1, a_2) \in A$, $b_1 \in A_1$, then there exists a $b_2 \in A_2$ such that $a \sim (b_1, b_2)$. A comparable statement can be made for Factor A_2.

Very simply, solvability means that any change in one factor (increase or decrease) can be exactly compensated for by a change (increase or decrease) in the second factor, producing an

"indifference" or "psychological equality" relation between any two elements (a_1, a_2) and (b_1, b_2).

It is important to note that the solvability axiom in this unrestricted form cannot, unlike transitivity, be directly tested empirically. Since our experimental design utilizes factors that would necessarily require finite levels, it would most likely not be possible to empirically solve inequalities exactly. This would not, of course, imply rejection of the axiom since other levels of the factors, not included in the study, might lead to solvability.

The third axiom is a <u>cancellation</u> axiom that is stated formally as

<u>Axiom 3</u>. Given (a_1, a_2), (b_1, b_2), and $(c_1, c_2) \in A$,

if $(a_1, b_2) \gtrsim (b_1, c_2)$ and

$(b_1, a_2) \gtrsim (c_1, b_2)$ then this implies

$(a_1, a_2) \gtrsim (c_1, c_2)$.

Given an additive representation on factors A_1 and A_2, it is clear that this axiom must hold. If we were to add the two antecedent inequality conditions, the b_1 and b_2 terms would cancel leaving the relation between (a_1, a_2) and (c_1, c_2). Hence, the property has become known as <u>double cancellation</u>.

Axiom 4 is another technical axiom that is common to many measurement systems because it is a fundamental property of the real number system. Before presenting this <u>Archimedean axiom</u> we first define a <u>dual standard sequence</u> (dss). Formally, we have

Axiom 4. $\{(a_{1i}, a_{1j}) \mid i, j = 0, \pm 1, \pm 2, \ldots\}$ in A is called a dss if $(a_{1i}, a_{2j}) \sim (a_{1k}, a_{2l})$ if and only if $i+j = k+l$.

If $\{(a_{1i}, a_{2j})\}$ is a dss, then for any $a \in A$, there exist integers m and n such that $(a_{1n}, a_{2n}) \gtrsim a \gtrsim (a_{1m}, a_{2m})$.

Axiom 4 indicates that no element in the set is either infinitely greater than or infinitely smaller than any other element in the set. Clearly, since this holds in the real number system (i.e., no number is infinitely bigger or smaller than any other number), it must hold in our empirical system if we hope to obtain numerical measurement scales.

Given this set of four axioms, Luce and Tukey (1964) were able to prove the following fundamental theorem of additive conjoint measurement.

THEOREM: If $\langle A_1, A_2, \gtrsim \rangle$ is an empirical relational system which satisfies Axioms 1-4, then, there exist real-valued functions ϕ on A, ϕ_1 on A_1, and ϕ_2 on A_2 such that for all (a_1, a_2) and (b_1, b_2) in A

(i) $(a_1, a_2) \gtrsim (b_1, b_2)$ iff $\phi(a_1, a_2) \geq \phi(b_1, b_2)$,

(ii) $\phi(a_1, a_2) = \phi_1(a_1) + \phi_2(a_2)$,

(iii) if ϕ^*, ϕ_1^*, and ϕ_2^* are any other functions which satisfy (i) and (ii) above, then there exist real num-

bers $\alpha > 0$, β_1, and β_2 such that

$\phi'_1 = \alpha * \phi_1 + \beta_1$,

$\phi'_2 = \alpha * \phi_2 + \beta_2$, and

$\phi' = \alpha * \phi + \beta_1 + \beta_2$.

It is important to recognize the power inherent in this theorem. Given four simple axioms that require only ordinal properties in the data for the binary relation $\succsim$, we arrive at a theorem which guarantees the existence of functions ϕ, ϕ_1, and ϕ_2 such that numerical scale values can be assigned to the stimulus objects in such a way that (1) the order among objects is preserved, (2) the levels of the factors on which the stimuli vary combine in an independent and additive fashion, and (3) the numerical scales have interval properties. Note that in (iii) above we are free to set the zero point of each scale, but the same unit value is applied to ϕ, ϕ_1, and ϕ_2. We will show later how some mathematical psychologists have developed procedures for actually constructing the scales ϕ, ϕ_1, and ϕ_2 on the basis of only the ordinal relationships among the stimuli in a given set of data.

Given the above presentation of the foundations of the two factor additive model, we can now proceed with a discussion of the more interesting three factor simple polynomial models as discussed by Krantz and Tversky (1971). There are four simple models that will be discussed. They are the familiar additive model

$$\phi(a_1, a_2, a_3) = \phi_1(a_1) + \phi_2(a_2) + \phi_3(a_3), \quad (2)$$

the multiplicative model

$$\phi(a_1, a_2, a_3) = \phi_1(a_1) * \phi_2(a_2) * \phi_3(a_3), \quad (3)$$

the distributive model

$$\phi(a_1, a_2, a_3) = \phi_1(a_1) * [\phi_2(a_2) + \phi_3(a_3)], \quad (4)$$

and the dual-distributive model

$$\phi(a_1, a_2, a_3) = \phi_1(a_1) + [\phi_2(a_2) * \phi_3(a_3)]. \quad (5)$$

Krantz and Tversky (1971) have previously discussed a number of ordinal properties that are necessary though not sufficient for these four models to hold. Since these properties form the basis of two computer programs that are used as diagnostic methods, they will be briefly summarized here. The intent of the Krantz and Tversky (1971) paper was not to present an axiomatization for each of the four models in Equations 2-5, but rather to describe a set of ordinal properties that may be used as diagnostic tools in differentiating among these four models as viable composition rules.

We begin with the fundamental property of <u>independence</u> which can be checked separately for each of the three factors. We say that

A_1 is <u>independent</u> of A_2 and A_3 whenever

$(a_1, a_2, a_3) \gtrsim (b_1, a_2, a_3)$ if and only if

$(a_1, b_2, b_3) \gtrsim (b_1, b_2, b_3)$. (6)

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Thus independence of A_1 asserts that if $a_1 > b_1$ for some combination of levels of factors A_2 and A_3, then this relation will hold for any other combination of levels of A_2 and A_3. Every test of independence of A_1 with A_2 and A_3 requires a 2x2x2 matrix with two levels of factor A_1 and two combinations of $A_2 \times A_3$. Thus the total number of possible tests of the property in this case would then be

$$T = \binom{n_1}{2} * \binom{n_2 * n_3}{2} \qquad (7)$$

where n_i is the number of levels of Factor i.

Although this property is clearly necessary for an additive model, it need not hold for any of the three models in Equations 3 - 5. This is because these latter models have multiplicative factors which might not preserve the order if negative or zero scale values are allowed. If all scale values for multiplicative factors are positive, however, the ordering of the stimuli cannot be reversed without violating the property. If a zero value is permitted for a multiplicative factor, then a degenerate case is produced regardless of the levels of the other factor(s). If negative values are permitted then a legitimate order reversal may occur. Hence, if only positive values are permitted, the independence property is necessary for all four models. If zero or negative values are permitted then we must define a more general property labelled sign dependence by Krantz and Tversky (1971).

A_1 is said to satisfy sign dependence if the other factors

A_2 and A_3 can be jointly partitioned into three sets $S^+ = |A_2 \times A_3|^+$, $S^0 = |A_2 \times A_3|^0$, and $S^- = |A_2 \times A_3|^-$ such that

(i) $(a_1, |a_2, a_3|^+) \gtrsim (b_1, |a_2, a_3|^+)$ if and only if

$$(a_1, |b_2, b_3|^+) \gtrsim (b_1, |b_2, b_3|^+), \tag{8}$$

(ii) $(a_1, |a_2, a_3|^-) \gtrsim (b_1, |a_2, a_3|^-)$ if and only if

$$(a_1, |b_2, b_3|^-) \gtrsim (b_1, |b_2, b_3|^-), \tag{9}$$

(iii) $(a_1, |a_2, a_3|^+) \gtrsim (b_1, |a_2, a_3|^+)$ if and only if

$$(a_1, |b_2, b_3|^-) \gtrsim (b_1, |b_2, b_3|^-), \tag{10}$$

(iv) $$(a_1, |a_2, a_3|^0) = (b_1, |a_2, a_3|^0). \tag{11}$$

In other words, if we restrict our comparisons in Equation 6 to combinations within the partitions S^+, S^0, and S^- (i.e., combinations of $A_2 \times A_3$ with the same "sign"), then independence should hold for all of our models. Equation 6 is then a special case of sign dependence when S^0 is empty and only one of either S^+ or S^- is nonempty.

A second form of independence can also be examined in our three-factor models. The property, known as <u>joint independence</u>, states that

A_1 and A_2 are <u>jointly independent</u> of A_3 whenever

$(a_1,a_2,a_3) \gtrsim (b_1,b_2,a_3)$ if and only if

$(a_1,a_2,b_3) \gtrsim (b_1,b_2,b_3)$. (12)

Joint independence of A_1 and A_2 with respect to A_3 indicates that if one combination of A_1 and A_2 is greater than another at a fixed level of A_3, (i.e., $(a_1,a_2) > (b_1,b_2)$ at a_3), then the ordering should be preserved for any other level of the third factor (b_3). If joint independence holds for all pairs of factors, then this implies that independence holds for a single factor. However, the converse is not necessarily true. If simple independence holds for all factors, this does not imply that joint independence will be satisfied for all pairs of factors.

We can, of course, state two other forms to the joint independence property for A_1 and A_3 of A_2, and A_2 and A_3 of A_1. If we again restrict our scale values for all factors to be positive, then it is clear that joint independence must hold in all three forms for the additive and multiplicative models. However, for the distributive model of the form $A_1*(A_2 + A_3)$ only A_2 and A_3 must be jointly independent of A_1. For any given set of finite observations, it is important to note that all three forms of joint independence may hold even if the model is, in fact, distributive. However, the larger the design, the more likely it is that only the one appropriate form will hold if the model is truely distributive.

It should be clear that we again are faced with the same potential problem discussed above for single factor independence -- namely, that zero or negative values will affect the orderings

of the stimuli in the joint independence property. We again must generalize the property by partitioning in this case the third factor A_3 into three sets $S^+ = A_3^+$, $S^0 = A_3^0$, and $S^- = A_3^-$ of the same "sign."

Joint sign dependence of A_1 and A_2 with A_3 is stated as

(i) $(a_1,a_2,a_3^+) \gtrsim (b_1,b_2,a_3^+)$ if and only if

$$(a_1,a_2,b_3^+) \gtrsim (b_1,b_2,b_3^+). \tag{13}$$

(ii) $(a_1,a_2,a_3^-) \gtrsim (b_1,a_2,a_3^-)$ if and only if

$$(a_1,a_2,b_3^-) \gtrsim (b_1,b_2,b_3^-). \tag{14}$$

(iii) $(a_1,a_2,a_3^+) \gtrsim (b_1,b_2,a_3^+)$ if and only if

$$(a_1,a_2,b_3^-) \gtrsim (b_1,b_2,b_3^-). \tag{15}$$

(iv) $$(a_1,a_2,a_3^0) = (b_1,b_2,a_3^0). \tag{16}$$

Thus, the property of joint independence is a special case of sign dependence where only S^+ or S^- is nonempty.

The third property examined by Krantz and Tversky (1971) is one that has already been discussed with respect to the Luce-Tukey axiomatization for the two-factor additive model. This is the property usually referred to as double cancellation or Luce-Tukey cancellation and is stated for factors A_1 and A_2 as

If $(a_1,b_2,a_3) \gtrsim (b_1,c_2,a_3)$ and

$(b_1,a_2,a_3) \gtrsim (c_1,b_2,a_3)$ then,

$(a_1,a_2,a_3) \gtrsim (c_1,c_2,a_3)$. (17)

Note that double cancellation requires at least three levels of each of factors A_1 and A_2, and deals with only two such factors at a time. Hence, it must be satisfied for all pairs of factors for any of the four models listed in Equations 2-5 when the scale values are all positive. If Factors A_1 and A_2 have n_1 and n_2 levels respectively, then there will be

$$T = \binom{n}{3}1) * \binom{n}{3}2) \quad (18)$$

possible tests of double cancellation for these two factors.

Like independence and joint independence, double cancellation need not hold in the signed case. Although it will not be presented in detail here, it can be shown (cf., Krantz and Tversky, 1971) that double cancellation will hold when subsets of the factors are established according to their "sign" as was done above for independence and joint independence.

Up to this point we have not presented a means of distinguishing between the distributive and dual-distributive models. The final two properties attempt to do this. We first describe a property known as <u>distributive cancellation</u>.

<u>Distributive cancellation</u> is satisfied if and only if

$(a_1,b_2,a_3) \gtrsim (d_1,c_2,c_3)$,

$(b_1,a_2,a_3) \gtrsim (c_1,d_2,c_3)$, and

$(d_1,d_2,c_3) \gtrsim (b_1,b_2,a_3)$, then

$(a_1,a_2,a_3) \gtrsim (c_1,c_2,c_3)$. (19)

It can be shown that this property is a necessary condition for the distributive model to hold. However, distributive cancellation also holds in an additive representation. Hence, although this property can be used to support a distributive representation, it cannot be used to reject additivity. It is not necessary for a dual-distributive representation, however, and can be used as a means to differentiate between these two models.

The final property to be discussed for our three-factor models is <u>dual-distributive cancellation</u>. Formally, we say that

If A_1 and A_2 are sign dependent on one another, then

<u>dual-distributive cancellation</u> is satisfied if and only if

$(a_1,c_2,c_3) \gtrsim (c_1,d_2,b_3)$,

$(a_1,e_2,e_3) \gtrsim (d_1,b_2,e_3)$,

$(d_1,a_2,a_3) \gtrsim (b_1,e_2,d_3)$,

$(d_1,c_2,d_3) \gtrsim (e_1,d_2,a_3)$, and

$(c_1,e_2,e_3) \gtrsim (e_1,b_2,e_3)$, then

$(a_1,a_2,b_3) \gtrsim (b_1,b_2,c_3)$. (20)

Dual-distributive cancellation is comparable to distributive cancellation in that it is necessary for both dual-distributive and additive representations. Hence, again it cannot be used to reject additivity. Since it is not necessary for a distributive representation, however, it can be used as a means of possibly distinguishing between a distributive and dual-distributive model. Note, however, that this property is extremely complex. It requires that five antecedent conditions from a 5x5x5 design be met in order for a test to even be possible. Hence, this property suffers from being empirically very difficult to evaluate.

Given this set of conditions it should be possible to evaluate each of the four polynomial models in Equations 2-5 for a set of observations obtained from a factorial design. In each of the axiom conditions only ordinal information is required in order to adequately test these properties. Thus it is sufficient to require each subject to merely present rank order judgments for each of the stimulus combinations generated by combining levels of the factors. As was discussed earlier, in most applications of conjoint measurement methodology it is the additive representation with restriction to the positive case that is of interest. However, even for an additive model as small as 3x3x3, both the testing procedures for the properties mentioned above and the actual scaling procedure for obtaining the numerical scale values become extremely impractical without the aid of a computer based algorithm. In the next two sections we will discuss several of the computer programs that were designed to do the testing and the scaling. We begin with a discussion of CONJOINT and PCJM, two programs designed to test the properties in Equations 6-20.

III. AXIOM TESTING PROGRAMS: CONJOINT AND PCJM2

One attempt to develop a general diagnostic program for testing the conjoint measurement axioms was made by Holt and Wallsten (1974). Their program, CONJOINT, was designed to test each of the axioms mentioned above except for dual-distributive cancellation. CONJOINT was written in PL/1 and has been modified to run on an IBM 370 or Amdahl 470 operating system. Ullrich and Cummins (1973) developed two other programs, PCJM and PCJM2, written in FORTRAN to do essentially the same thing as CONJOINT. There are, however, several differences between the programs which make both useful as diagnostic tools. In the following sections, both CONJOINT and PCJM2 will be discussed with respect to their input and output.

Table 1

Rankings of Riskiness from CONJOINT

for a 4x4x3 Design

C	A	1	2	3	4 (B)
1	1	47.0	42.5	36.5	29.0
1	2	42.5	29.0	14.5	10.0
1	3	36.5	14.5	7.5	4.5
1	4	29.0	10.0	4.0	3.5
2	1	47.0	42.5	36.5	29.0
2	2	42.5	29.0	22.5	18.5
2	3	36.5	22.5	13.5	5.5
2	4	29.0	14.0	4.0	3.5
3	1	46.5	42.5	36.5	29.0
3	2	43.0	29.0	22.5	18.5
3	3	36.5	22.5	16.5	12.5
3	4	29.0	18.5	12.5	3.5

The data shown in Table 1 will be used to illustrate the applications of the programs discussed in the remaining sections of this report. The values in the table are rankings formed by the CONJOINT program. These data came from one subject's ratings of "riskiness" for each of 48 gambles from a 4x4x3x1 design (where A is "Amount to Lose", B is "Probability of Losing", C is "Probability of Winning", and "Amount to Win" is held constant). The original data from which these ranks were formed can be found on pages 7-8 of the CPSCAL manual.

Throughout the discussion of the foundations of conjoint measurement we have used the notation A_1, A_2, and A_3 to represent our factors. Two different ways to denote the factors will now be introduced. Although it may at first seem confusing to introduce this additional notation, it is necessary since these notational changes are used rather extensively in the CONJOINT and PCJM2 programs. CONJOINT uses the Roman letters A, B, and C to represent the factors (Tables 1-5) whereas PCJM2 uses the letters A, P, and U to represent the corresponding three factors (Tables 6-12).

CONJOINT

Both CONJOINT and PCJM2 allow one to test for independence among the factors, although the approach taken by each is somewhat different. CONJOINT tests for independence of factors by considering them two at a time. Independence for A of B would be checked by comparing the rank order of the cells for the levels of Factor A at each level of Factor B. Similarly, a check can be made for the independence of B at each level of A.

Table 2

Results of Tests of Simple Independence

from CONJOINT Analysis

FACTOR A IS THE OUTSIDE FACTOR.

	1	2	3	4
B of C	1.00	1.00	1.00	1.00
C of B	0.25	0.58	0.70	0.46

FACTOR B IS THE OUTSIDE FACTOR.

	1	2	3	4
A of C	1.00	1.00	1.00	1.00
C of A	0.00	0.46	0.63	0.46

FACTOR C IS THE OUTSIDE FACTOR.

	1	2	3
A of B	1.00	1.00	1.00
B of A	1.00	1.00	1.00

To illustrate the property of independence, let us look at the A x B matrix at fixed Level 1 of Factor C. Note that in comparing the rank orders of the four rows of this matrix, there is perfect agreement. Hence, we say "B is independent of A" at Level 1 of Factor C. It is also the case, however, that "B is independent of A" at Levels 2 and 3 of C (the second and third matrices), and we can say simply that "B is independent of A". In a comparable manner we can look at the ranks of the columns for the A x B matrix at each level of C. Again, we find perfect rank order agreement. Hence, we also say "A is independent of B". It is important to recognize that "A independent of B" does not imply nor is implied by "B independent of A". To illustrate this, suppose that the data value in cell (4,3,1) had been 2.0 rather than 4.0. Then, although independence of A from B would hold, independence of B from A would be violated. A second point to recognize is that we are only looking at independence for two factors at a time at this point.

Table 3

Results of Tests of Joint Independence from CONJOINT Analysis

```
FACTOR A IS THE OUTSIDE FACTOR.
         BC OF A    W=    0.9583
         A OF BC    W=    1.0000

FACTOR B IS THE OUTSIDE FACTOR.
         AC OF B    W=    0.9680
         B OF AC    W=    1.0000

FACTOR C IS THE OUTSIDE FACTOR.
         AB OF C    W=    0.9990
         C OF AB    W=    0.3063
```

For the data in Table 1, CONJOINT would produce the results for tests of independence shown in Table 2. The numbers in Table 2 are Kendall's Coefficients of Concordance (W) across rows or across columns at each level of the outside factor. Thus, if independence is satisfied, then the rows and columns in Table 1 should all be in the same rank order, yielding W values equal to 1.0. To the extent that some of the W values are near zero we may have either (1) nonindependence of factors, (2) degenerate level(s) of some factor or factors or (3) irrelevance of a factor. The last possibility is particularly interesting from an empirical standpoint. If one were to look only at the W values in Table 2, one might be tempted to conclude that no simple polynomial model fits the data. However, violations of independence occurred because the subject apparently did not differentiate among the levels of Factor C (probability of winning) as shown by the "rankings" in Table 1. The subject's judgments of risk appear to be based only on the two independent factors, probability of losing and amount to lose.

Table 3 presents similar results for joint independence. The W values are all very near 1.0, except for the test of "C of AB." To understand the implications of the tests of independence (Table 2) and joint independence (Table 3), it is important to follow how the W-values were computed. First, we will illustate simple independence with the value of W = .58 from Table 2. This value was obtained from the check of independence for C of B at level A_2. It was obtained by comparing the rank orders of the following four sets (B_1-B_4) of three numbers (C_1-C_3):

(1)	42.5,	42.5,	43.0
(2)	29.0,	29.0,	29.0
(3)	14.5,	22.5,	22.5
(4)	10.0,	18.5,	18.5

In a comparable manner independence for B of C at level A where W = 1.00 was obtained by comparing the rank orders of the three sets (C_1-C_3) of four numbers (B_1-B_4):

(1)	42.5,	29.0,	14.5,	10.0
(2)	42.5,	29.0,	22.5,	18.5
(3)	43.0,	29.0,	22.5,	18.5

The joint independence value of W = .3063 from Table 3 for "C of AB" was obtained by comparing the rank orders of the following 16 sets (A_1,B_1 ; A_1,B_2 ; ... ; A_4,B_4) of three numbers (C_1-C_3):

(1)	47.0,	47.0,	46.5
(2)	42.5,	42.5,	42.5
(3)	36.5,	36.5,	36.5
.			
.			
.			
(15)	4.0,	4.0,	12.5
(16)	3.5,	3.5,	3.5

Finally, W = .9990 for "AB of C" was found from the ranks of three sets of 16 numbers:

(1)	47.0,	42.5,	36.5,	29.5,	... ,	10.0,	4.0,	3.5
(2)	47.0,	42.5,	36.5,	29.5,	... ,	14.0,	4.0,	3.5
(3)	46.5,	42.5,	36.5,	29.0,	... ,	18.5,	12.5,	3.5

In addition to the independence tests, CONJOINT allows one to test both Luce-Tukey or double cancellation and distributive cancellation. Tests of these properties are illustrated for the risk data in Tables 4 and 5. Double cancellation is checked for each pair of factors at each level of the third ("outside") factor. Table 4 illustrates one such test -- Factors A and B at each level of Factor C. At each level of C there will be 16 possible tests. Recall that each test of double cancellation requires a 3x3 submatrix. With n = 4 levels of Factor A and n = 4 levels of Factor B, there will be 16 possible tests.

Table 4

Results of Tests of Double Cancellation from CONJOINT

Analysis: Factors A and B at Each Level of Factor C

```
L-T CANCELLATION
FACTOR C IS THE OUTSIDE FACTOR.

LEVEL C 1      2 VIOLATIONS.
 0 TESTS NOT POSSIBLE.
14 SUCCESSFUL TESTS.

LEVEL C 2      2 VIOLATIONS.
 0 TESTS NOT POSSIBLE.
14 SUCCESSFUL TESTS.

LEVEL C 3      2 VIOLATIONS.
 0 TESTS NOT POSSIBLE.
14 SUCCESSFUL TESTS.
```

For the data in Table 4 all 16 tests at each level of Factor C were possible. The "Tests Not Possible" counter would have been

nonzero only if the antecedent conditions in Equation 17 had not been met for one or more of the 3x3 submatrices. Given that a test was possible in each submatrix, the program then checked for violations. A violation occurs whenever either or both of the antecedent conditions in Equation 17 produce a strict inequality in one direction, but the consequence in an equality or strict inequality in the opposite direction. As an example, let us find one of the two violations for Factors A and B at Level C . One violation occurs in the 3x3 submatrix below with Levels A_2-A_4 and B_2-B_4. Note that $(a_3,b_2) \sim (a_2,b_3)$ and $(a_4,b_3) < (a_3,b_4)$,

	B2	B3	B4
A2	29.0	14.5	10.0
A3	14.5	7.5	4.5
A4	10.0	4.0	3.5

but that $(a_4,b_2) \sim (a_2,b_4)$. The consequence cannot be an equality because one of the antecedent conditions was a strict inequality. It is important to recognize from this example that there are several (in fact, there are nine) ways to distinctly permute the rows and columns of this specific (or any) 3x3 submatrix in order to test for double cancellation. CONJOINT automatically does this. Note that there is a second test that is actually satisfied by the data in this matrix; it is $(a_3,b_2) > (a_4,b_3)$ and $(a_2,b_3) > (a_3,b_4)$ imply $(a_2,b_2) > (a_4,b_4)$. However, this check of the matrix is not counted as a "successful" test; CONJOINT still counts this 3x3 matrix as producing a violation since one (or more) of the nine possible tests has failed.

Table 5 illustrates the results of tests of distributive cancellation for Factors A and B at pairs of levels of Factor C.

Table 5

Results of Tests of Distributive Cancellation from CONJOINT Analysis: Factors A and B At Each Level of C

```
DISTRIBUTIVE CANCELLATION
FACTOR C IS THE OUTSIDE FACTOR.

LEVELS                          1     VS.     2
          18    VIOLATIONS
          88    TESTS NOT POSSIBLE
        1190    SUCCESSFUL TESTS

LEVELS                          1     VS.     3
          68    VIOLATIONS
          41    TESTS NOT POSSIBLE
        1187    SUCCESSFUL TESTS

LEVELS                          2     VS.     3
          65    VIOLATIONS
          75    TESTS NOT POSSIBLE
        1156    SUCCESSFUL TESTS
```

Comparable tests were done for Factors A and C for pairs of levels of Factor B and for B and C for pairs of levels of A. From Equation 19 it is clear that to test this axiom we need a 2x2 matrix in the AB plane at two different levels of Factor C. Holt and Wallsten (1974) have shown that there are 16 permutations of the pair of 2x2 matrices that define distributive cancellation. For each pair of these 2x2 matrices CONJOINT will test all 16 permutations if necessary. In a manner comparable to that of double cancellation, the program will record a failure for the 2x2 matrix if any one of the 16 tests fails. If the antecedent conditions are not met in any of the 16 tests, then the "Tests Not Possible" counter is incremented.

Since all 2x2 matrices for A and B are checked at any two levels of C, there will be

$$n \quad n \quad n \quad n$$

$$T = \binom{6}{2} * \binom{6}{2} * \binom{6}{2} * \binom{6}{2} \quad (21)$$

or 36 x 36 = 1296 such tests for the risk rating data for each level of C1 vs. C2, C1 vs. C3, and C2 vs. C3. The actual number of violations, tests not possible, and successful tests for the risk data are shown in Table 5. CONJOINT would also print the results of comparable tests for A and C with B as the outside factor and for B and C with A as the outside factor. In this way one could check the feasibility of each of the distributive models A*(B+C), B*(A+C), and C*(A+B). In addition, even if the data were proposed to fit an additive model, these tests would still be very useful since distributive cancellation would have to hold in all three forms for an additive representation.

There is one final note worth mentioning about distributive cancellation. It appears from extended tests of the distributive cancellation axiom across many data sets that this axiom is not very powerful as a diagnostic tool. That is, it appears that even for data that have a high degree of error variance associated with them, distributive cancellation will be supported most of the time. CONJOINT does print one additional piece of information that may be useful to the researcher, however. After printing the results of the tests of distributive cancellation, the program prints a matrix indicating the number of times each cell was involved in a violation. An illustration of this matrix will be shown when the comparable PCJM2 procedure is discussed. Sometimes systematic violations may aid in identifying particular problem levels of the factors.

Table 6

Ranked Risk Ratings from PCJM2 Analysis

for a 4x4x3 Design

DATA MATRIX BEING CHECKED FOR INDEPENDENCE:

MATRIX ID NO. = 1 U = 1

A =	1	2	3	4
P = 1	2.00	6.50	12.50	20.00
P = 5	6.50	20.00	34.50	39.00
P = 9	12.50	34.50	41.50	45.00
P =13	20.00	39.00	44.50	45.50

MATRIX ID NO. = 1 U =17

A =	1	2	3	4
P = 1	2.00	6.50	12.50	20.00
P = 5	6.50	20.00	26.50	34.50
P = 9	12.50	26.50	35.50	45.00
P =13	20.00	30.50	43.50	45.50

MATRIX ID NO. = 1 U =33

A =	1	2	3	4
P = 1	2.50	6.00	12.50	20.00
P = 5	6.50	20.00	26.50	30.50
P = 9	12.50	26.50	32.50	36.50
P =13	20.00	30.50	36.50	45.50

PCJM2

Let us now examine the same riskiness data using the PCJM2 program. The rankings of the 48 data cells in the 4x4x3 design are presented again in Table 6. Notice that the values in this table are the complements of the rankings of 1 to 48 from Table 1. In general, PCJM2 ranks the data in the opposite direction of CONJOINT when there are multiple observations per cell. In the illustrative data used here there were three entries in each of the 48 data cells. That is, the subject gave three ratings of riskiness to each of the 48 gambles. In situations such as this with multiple observations in each cell, we could proceed in either of two ways to obtain ranks for the cells. One method would be to simply average the observations in each cell and then

rank these averages. A more statistically useful method was used by Holt and Wallsten (1974). If there are three or more observations per cell then CONJOINT proceeds as follows. Each cell is compared with every other cell via a Mann-Whitney U-test. If the U-value for a comparison is greater than the critical value at the .05 significance level, then the two cells are considered unequal in rank. If the U-value is not significant, the two cells are considered equal. A counter is employed to keep a record of all cells that a particular cell exceeds. The rank for the cell is given by the number of cells that it has exceeded plus one-half the number of cells it has tied. Exactly the same procedure is used by PCJM2 except that the rank of a cell is equal to the number of cells that it is smaller than plus one-half the number it has tied. Thus the values in Table 6 are equal to 48 minus the corresponding value in Table 1, plus one.

It should not be surprising that a number of the tied cells were formed from the present data set. With only three observations in each cell, it can be shown that only when all three values of one cell exceed the values from another cell that a significant U-value at the .05 level of significance can be obtained. With only two observations in each cell, a significant U-value could not be obtained at the .05 level. Hence, both CONJOINT and PCJM2 use this Mann-Whitney U-test only when the number of replications in the cells is equal to or greater than three. If there are only two observations in each cell, the researcher must first alter the data to obtain the ranks. Perhaps here a simple averaging procedure would be best. When there is only one observation per cell, neither program makes any

Table 7

Results of Tests of Simple Independence

from PCJM2 Analysis

```
TEST FAILURE u= 1 AND v=33 ARE REVERSED AT a=1 p=1 AND b=2 q= 9
TEST FAILURE u= 1 AND v=33 ARE REVERSED AT a=1 p=1 AND b=2 q=13
TEST FAILURE u= 1 AND v=33 ARE REVERSED AT a=1 p=1 AND b=3 q= 5
TEST FAILURE u= 1 AND v=33 ARE REVERSED AT a=1 p=1 AND b=3 q= 9
TEST FAILURE u=17 AND v=33 ARE REVERSED AT a=1 p=1 AND b=3 q= 9
TEST FAILURE u= 1 AND v=33 ARE REVERSED AT a=1 p=1 AND b=3 q=13
TEST FAILURE u=17 AND v=33 ARE REVERSED AT a=1 p=1 AND b=3 q=13
TEST FAILURE u= 1 AND v=33 ARE REVERSED AT a=1 p=1 AND b=4 q= 5
TEST FAILURE u=17 AND v=33 ARE REVERSED AT a=1 p=1 AND b=4 q= 5
TEST FAILURE u= 1 AND v=33 ARE REVERSED AT a=1 p=1 AND b=4 q= 9
ETC.

TEST SUMMARY STATISTICS:
TOTAL POSSIBLE TESTS ("NUMBER TESTS"), NON-TESTS IN THE DATA
("NUMBER EQUALS"), AND TOTAL ERRORS ("NUMBER FAILS").

   NUMBER TESTS   A INDEPT  P X U    396
   NUMBER EQUALS  A INDEPT  P X U      0
   NUMBER FAILS   A INDEPT  P X U      0
   AVERAGE TAU    A INDEPT  P X U  1.000

   NUMBER TESTS   P INDEPT  U X A    396
   NUMBER EQUALS  P INDEPT  U X A      1
   NUMBER FAILS   P INDEPT  U X A      0
   AVERAGE TAU    P INDEPT  U X A  0.997

   NUMBER TESTS   U INDEPT  A X P    360
   NUMBER EQUALS  U INDEPT  A X P    296
   NUMBER FAILS   U INDEPT  A X P     11
   AVERAGE TAU    U INDEPT  A X P  0.117
```

modification to the data since only the ordinal relations among the values are used. Finally, PCJM2 and CONJOINT do not require every data cell to be non-empty (or to have three or more observations in <u>every</u> cell in the case of multiple entry data). Both programs will handle missing data designs, although CONJOINT will not run if 10% or more of the cells are empty.

Table 8

Distribution of Failures in Simple Independence for Each Cell of the Data Matrix

MATRIX ID NO. = 1 U = 1

A =	1	2	3	4
P = 1	7.00	0.0	0.0	0.0
P = 5	0.0	0.0	1.00	1.00
P = 9	0.0	1.00	1.00	1.00
P = 13	0.0	1.00	1.00	0.0

MATRIX ID NO. = 1 U = 17

A =	1	2	3	4
P = 1	4.00	0.0	0.0	0.0
P = 5	0.0	0.0	0.0	1.00
P = 9	0.0	0.0	1.00	1.00
P = 13	0.0	0.0	1.00	0.0

MATRIX ID NO. = 1 U = 33

A =	1	2	3	4
P = 1	11.00	0.0	0.0	0.0
P = 5	0.0	0.0	1.00	2.00
P = 9	0.0	1.00	2.00	2.00
P = 13	0.0	1.00	2.00	0.0

Tables 7 and 8 present the results of the PCJM2 analysis of the riskiness data for tests of independence. As was mentioned earlier, PCJM2 actually tests for independence in the manner suggested by Krantz and Tversky (1971) and stated in Equation 6. From Equation 7 it is clear that for the riskiness data there should be 6x66 = 396 tests of independence for each of Factors A and P, and 3x120 = 360 tests for Factor U. Let us look at the 360 possible tests of "U independent of A and P." Notice that here 296 of the tests could not be done ("NUMBER EQUALS" = 296). A non-test of independence is counted by PCJM2 when one of the two conditions in Equation 6 is an equality. An error is counted when one condition has an inequality in one direction and the second has an inequality in the other direction. Table 7 is again indicating that Factor U was apparently irrelevant to the subject in making riskiness ratings. Hence, while violations of the axiom

are clearly important diagnostic indices of what is happening in the data, so are the number of non-tests.

Table 9

Results of Tests of Joint Independence

from PCJM2 Analysis

```
DATA MATRIX BEING CHECKED FOR JOINT INDEPENDENCE:
TEST FAILURE: p= 5 x u= 1 IS NOT INDEPENDENT OF a= 1 AND b= 3
TEST FAILURE: p= 5 x u= 1 IS NOT INDEPENDENT OF a= 1 AND b= 4
TEST FAILURE: p= 5 x u= 1 IS NOT INDEPENDENT OF a= 2 AND b= 3
TEST FAILURE: p= 5 x u= 1 IS NOT INDEPENDENT OF a= 2 AND b= 4
TEST FAILURE: p= 5 x u= 1 IS NOT INDEPENDENT OF a= 1 AND b= 2
TEST FAILURE: p= 5 x u= 1 IS NOT INDEPENDENT OF a= 2 AND b= 3
TEST FAILURE: p= 5 x u= 1 IS NOT INDEPENDENT OF a= 2 AND b= 4
TEST FAILURE: p= 5 x u= 1 IS NOT INDEPENDENT OF a= 1 AND b= 2
TEST FAILURE: p= 5 x u= 1 IS NOT INDEPENDENT OF a= 1 AND b= 3
TEST FAILURE: p= 5 x u= 1 IS NOT INDEPENDENT OF a= 2 AND b= 4
ETC.

TEST SUMMARY STATISTICS:
TOTAL POSSIBLE TESTS ("NUMBER TESTS"), NON-TESTS IN THE DATA
("NUMBER EQUALS"), AND TOTAL ERRORS ("NUMBER FAILS").

NUMBER OF TESTS  FOR  AXP:U   360     PXU:A   396     UXA:P   396
NUMBER OF EQUALS FOR  AXP:U    25     PXU:A    64     UXA:P    62
NUMBER OF FAILS  FOR  AXP:U     0     PXU:A    11     UXA:P     8
AVERAGE TAU      FOR  AXP:U 0.9306    PXU:A 0.7828    UXP:A 0.8030
```

Also of value in examining independence among the factors (are obtainable from PCJM2) are (1) the average value of Kendall's Tau for ordering the levels of the factors, (2) a list of violations, and (3) a matrix of the distribution of violations of the axiom. Unlike CONJOINT, PCJM2 computes a Kendall's Tau coefficient in comparing the ranks of the cells for all pairs of levels of each factor. The average of these Taus is computed and printed for each factor. In the example in Table 7, Factor A had no violations or non-tests, so the average Tau was 1.00. Factor B

has an average Tau of slightly less than 1.00 (Tau = .997) since there was only one non-test (i.e., one tied rank) in the data. Factor U had a low average Tau because of the number of violations and non-tests in the data for that factor.

One may specify when using the PCJM2 program that a number of the violations are to be printed. In this example the request was to list up to ten violations for each axiom. If more than the requested number of violations to be printed have actually occurred, the program indicates this by writing "ETC." following the last printed violation.

Table 10

Distribution of Failures of Joint Independence
for Each Cell of the Data Matrix from the PCJM2 Analysis

MATRIX ID NO. = 1 U = 1

A =	1	2	3	4
P = 1	0.0	2.00	2.00	0.0
P = 5	2.00	4.00	4.00	2.00
P = 9	3.00	7.00	5.00	3.00
P =13	0.0	2.00	2.00	0.0

MATRIX ID NO. = 1 U =17

A =	1	2	3	4
P = 1	0.0	0.0	0.0	0.0
P = 5	0.0	0.0	0.0	0.0
P = 9	0.0	0.0	0.0	0.0
P =13	2.00	4.00	0.0	0.0

MATRIX ID NO. = 1 U =33

A =	1	2	3	4
P = 1	0.0	0.0	4.00	4.00
P = 5	0.0	0.0	4.00	2.00
P = 9	4.00	4.00	0.0	2.00
P =13	4.00	2.00	2.00	0.0

Using the PCJM2 notation of a,b,c..., p,q,r..., and u,v,w...

to denote the levels of the factors, we can illustrate the first violation of independence listed in Table 7. We have (1,1,1) < (1,1,33) but (2,9,1) > (2,9,33). That is, from the rankings: 2.00 < 2.50 but 34.5 > 26.5. The other violations can be interpreted in a comparable manner. Finally, the summary matrix in Table 8 shows how many times each AxPxU cell was involved in a violation.

Tables 9, 10, 11, and 12 illustrate the PCJM2 analysis of the riskiness data for the joint independence, double cancellation, and distributive cancellation properties. Although the program can test for dual-distributive cancellation, the illustrative data set used here is not large enough for appropriate tests of this axiom. Tables 9 and 10 show for joint independence the information concerning tests and violations of the axioms as presented in Equation 12. The program will list (1) as many violations of the axiom as indicated by the user, (2) the number of possible tests, errors, and non-tests, and (3) the average Tau value for each pair of factors. Table 10 indicates the number of violations associated with each cell of the matrix. Tables 11 and 12 illustrate double and distributive cancellation. For both of these cancellation axioms, PCJM2 will summarize the number of violations, the actual failure locations, and the summary matrix of cells involved in violations. Since there were no violations of double cancellation in this data, however, only the summary statistics were presented.

Table 11

Results of Tests of Double Cancellation and Distributive Cancellation from the PCJM2 Analysis

```
DATA MATRIX BEING CHECKED FOR DOUBLE CANCELLATION:
TEST SUMMARY STATISTICS:
NUMBER OF TESTS FOR DBL CAN IN AXP = 38    PXU = 16    AXU = 16
NUMBER OF FAILS FOR DBL CAN IN AXP =  0    PXU =  0    AXU =  0


DATA MATRIX BEING CHECKED FOR DISTRIBUTIVE CANCELLATION:
FACTOR  1 IS THE OUTSIDE FACTOR.
TEST SUMMARY STATISTICS:
    NUMBER OF TESTS =  503      NUMBER OF FAILURES =  19

DATA MATRIX BEING CHECKED FOR DISTRIBUTIVE CANCELLATION:
FACTOR  2 IS THE OUTSIDE FACTOR.
TEST SUMMARY STATISTICS:
    NUMBER OF TESTS =  503      NUMBER OF FAILURES =  25

DATA MATRIX BEING CHECKED FOR DISTRIBUTIVE CANCELLATION:
FACTOR  3 IS THE OUTSIDE FACTOR.
TEST FAILURE AT RECTANGLES
a= 1 b= 2 p= 1 q= 9 AT u= 1  AND  c= 1 d= 2 r= 1 s=13 AT v=17
a= 3 b= 2 p= 1 q= 9 AT u= 1  AND  c= 3 d= 2 r= 1 s=13 AT v=17
a= 4 b= 2 p= 1 q= 9 AT u= 1  AND  c= 4 d= 2 r= 1 s=13 AT v=17
a= 1 b= 2 p= 5 q= 9 AT u= 1  AND  c= 2 d= 4 r= 1 s= 5 AT v=17
a= 1 b= 2 p= 5 q= 9 AT u= 1  AND  c= 1 d= 2 r= 5 s=13 AT v=17
a= 3 b= 2 p= 5 q= 9 AT u= 1  AND  c= 3 d= 2 r= 5 s=13 AT v=17
a= 3 b= 1 p= 5 q= 1 AT u= 1  AND  c= 4 d= 1 r= 5 s= 1 AT v=17
a= 3 b= 1 p= 5 q= 9 AT u= 1  AND  c= 4 d= 1 r= 5 s= 9 AT v=17
a= 3 b= 1 p= 5 q=13 AT u= 1  AND  c= 4 d= 1 r= 5 s= 9 AT v=17
a= 3 b= 1 p= 5 q=13 AT u= 1  AND  c= 4 d= 1 r= 5 s=13 AT v=17
ETC.

TEST SUMMARY STATISTICS:
    NUMBER OF TESTS =  2723          NUMBER OF FAILURES =  166
```

Table 12

Distribution of Failures of Distributive Cancellation for Each Cell of the Data Matrix from the PCJM2 Analysis

MATRIX ID NO. = 1 U = 1

A =	1	2	3	4
P = 1	12.00	38.00	25.00	23.00
P = 5	38.00	97.00	46.00	41.00
P = 9	21.00	42.00	19.00	20.00
P =13	25.00	41.00	24.00	24.00

MATRIX ID NO. = 1 U =17

A =	1	2	3	4
P = 1	17.00	32.00	16.00	21.00
P = 5	23.00	40.00	17.00	38.00
P = 9	11.00	17.00	9.00	19.00
P =13	19.00	35.00	14.00	24.00

MATRIX ID NO. = 1 U =33

A =	1	2	3	4
P = 1	11.00	40.00	18.00	31.00
P = 5	25.00	42.00	24.00	39.00
P = 9	12.00	25.00	12.00	25.00
P =13	26.00	33.00	26.00	51.00

IV. ADDITIVE CONJOINT SCALING PROGRAMS: NONMETRG AND MONANOVA

Let us assume that we have used the CONJOINT and PCJM2 programs with our data set and have found that an additive model appears to be supported. We may now want to obtain a scaling solution for the data such that the numerical values associated with the levels of the independent variables combine additively to form scale values for the stimulus combinations. In all but the simplest situations it is virtually impossible to attempt to simultaneously scale the independent and dependent variables using an additive (or any other) composition rule without the aid of a computer program. With the advent of computer-based algorithms originally intended for non-metric multidimensional scaling of similarities data (cf. Shepard, 1962a, 1962b; Kruskal, 1964a, 1964b; Johnson, 1973), it became possible to obtain conjoint scaling

solutions for even complex data sets. A number of algorithms are now available for obtaining these scaling solutions for either an additive, multiplicative, distributive or dual-distributive model. Four of these programs will be discussed here. The first two will deal with an additive representation; the latter two will be used with the distributive and dual-distributive models, respectively.

NONMETRG

NONMETRG is a modified version of a scaling procedure originally proposed by Johnson (1973). As the name of the program implies, NONMETRG is a non-metric scaling procedure. By that we mean that it attempts to find interval-scaled values for the levels of the independent variables based only on the ordinal relationships exhibited in the data. The objective of the program, as with all non-metric programs is simple: to use the ordinal constraints imposed on the data to find an effectively unique solution (i.e., unique up to an affine transformation). In a metric procedure we assume that the data are related to the final distance scale values by

$$d'_j = a * d_j + b \tag{22}$$

where d_j is the original response judgment for stimulus j, d'_j is the rescaled distance value, and a > 0 and b are constants. In a non-metric procedure, the function need only be monotonic. That is,

$$d'_j \overset{m}{=} f(d_j) \tag{23}$$

NONMETRG is different from most non-metric programs in that it does not directly attempt to find the monotonic transformation in Equation 23. The procedure employed by the program is a one-step pairwise method. This procedure will now be summarized.

NONMETRG, like many non-metric programs is an iterative technique. By this we mean that the program continues to search for a set of scale values that will "best" reproduce the ordinal relationships among the stimuli according to specified composition rule (e.g., an additive rule in our example). The search or iterative procedure continues until a cutoff criterion is achieved or until no practical improvements can be found. Hence, to begin the scaling process NONMETRG needs three sets of values. First, the program needs a set of parameters that describe the independent factors and the data to be rescaled. Next, the data is read in the form of an $nx(m+1)$ matrix where n represents the number of stimuli and m represents the total number of levels of all of the factors. In the illustrative risk-taking example, m would equal 4+4+3 = 11. The first m values in each row of the matrix are coded as 0's or 1's. A zero indicates the absence of the corresponding level of a factor and a one represents its presence. The last value in each row is the actual response judgment. Before doing the actual scaling, NONMETRG will print this input matrix so that the user may check for errors. Table 13 illustrates this printed input matrix for the riskiness data.

Ignoring the first two indexing columns of this matrix for now, we can see that there are m+1 = 12 columns remaining. The stimuli in this example are in the natural order; that is,

Table 13

Input Matrix from Riskiness Data

Used in the NONMETRG Analysis

```
TITLE:  CONJOINT SCALING:  RISKINESS DATA.
TITLE:  48  STIMULI.    1X4X4X3 DESIGN.
TITLE:  FACTORS ARE:
TITLE:  AMOUNT TO LOSE, 4 LEVELS.  -10, -20, -30, AND -40 CENTS.
TITLE:  PROBABILITY OF LOSING, 4 LEVELS.  1/8, 2/8, 3/8, AND 4/8.
TITLE:  PROBABILITY OF WINNING, 3 LEVELS.  2/8, 3/8, AND 4/8.
TITLE:  DATA ARE THREE RISKINESS RATINGS FOR EACH OF 48 GAMBLES.
TITLE:  SCALE RANGES FROM '1' TO '100'.
TITLE:  STIMULI ARE IN THE NATURAL ORDER.
```

DATA MATRIX: SUBJECT/REPLICATION NO. 1

BLOCK	STIM	LEVELS OF FACTORS											
1	1	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	3.0
1	2	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	4.0
1	3	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	4.0
1	4	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	9.0
1	5	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	[illegible]
1	6	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	[illegible]
1	7	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	14.0
1	8	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	15.0
1	9	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	15.0
1	10	1.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	19.0
1	11	1.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	19.0
1	12	1.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	19.0
1	13	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	9.0
1	14	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	9.0
1	15	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	6.0
1	16	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	19.0
1	17	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	18.0
1	18	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	19.0
1	19	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	52.0
1	20	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	29.0
1	21	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	29.0
1	22	0.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	89.0
1	23	0.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	38.0
1	24	0.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	39.0
1	25	0.0	0.0	1.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	15.0
1	26	0.0	0.0	1.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	15.0
1	27	0.0	0.0	1.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	14.0
1	28	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	51.0
1	29	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	29.0
1	30	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	29.0
1	31	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	92.0
1	32	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	61.0
1	33	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	46.0
1	34	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	96.0
1	35	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	95.0
1	36	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	59.0
1	37	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	19.0
1	38	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	19.0
1	39	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	19.0

1	40	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	88.0
1	41	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	39.0
1	42	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	39.0
1	43	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	94.0
1	44	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	96.0
1	45	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	59.0
1	46	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	98.0
1	47	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	97.0
1	48	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	98.0
2	1	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	5.0
2	2	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	5.0
2	3	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	5.0
2	4	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	9.0
2	5	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	9.0
2	6	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	9.0
2	7	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	15.0
2	8	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	15.0
2	9	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	15.0
2	10	1.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	19.0
2	11	1.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	19.0
2	12	1.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	19.0
2	13	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	9.0
2	14	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	9.0
2	15	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	9.0
2	16	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	19.0
2	17	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	19.0
2	18	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	19.0
2	19	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	52.0
2	20	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	29.0
2	21	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	28.0
2	22	0.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	89.0
2	23	0.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	39.0
2	24	0.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	39.0
2	25	0.0	0.0	1.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	15.0
2	26	0.0	0.0	1.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	14.0
2	27	0.0	0.0	1.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	14.0
2	28	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	52.0
2	29	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	29.0
2	30	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	29.0
2	31	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	91.0
2	32	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	51.0
2	33	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	45.0
2	34	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	96.0
2	35	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	94.0
2	36	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	59.0
2	37	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	19.0
2	38	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	19.0
2	39	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	19.0
2	40	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	89.0
2	41	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	39.0
2	42	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	39.0
2	43	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	96.0
2	44	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	93.0
2	45	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	59.0
2	46	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	99.0
2	47	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	98.0
2	48	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	97.0
3	1	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	5.0
3	2	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	5.0
3	3	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	6.0
3	4	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	9.0

3	5	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	9.0
3	6	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	9.0
3	7	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	15.0
3	8	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	15.0
3	9	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	15.0
3	10	1.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	19.0
3	11	1.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	19.0
3	12	1.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	19.0
3	13	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	9.0
3	14	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	9.0
3	15	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	9.0
3	16	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	19.0
3	17	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	19.0
3	18	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	19.0
3	19	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	51.0
3	20	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	29.0
3	21	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	29.0
3	22	0.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	89.0
3	23	0.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	39.0
3	24	0.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	39.0
3	25	0.0	0.0	1.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	16.0
3	26	0.0	0.0	1.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	15.0
3	27	0.0	0.0	1.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	15.0
3	28	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	51.0
3	29	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	29.0
3	30	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	29.0
3	31	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	91.0
3	32	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	52.0
3	33	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	45.0
3	34	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	92.0
3	35	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	94.0
3	36	0.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	59.0
3	37	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	19.0
3	38	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	19.0
3	39	0.0	0.0	0.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	1.0	19.0
3	40	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	89.0
3	41	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	89.0
3	42	0.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	0.0	0.0	1.0	39.0
3	43	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	1.0	0.0	0.0	95.0
3	44	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	94.0
3	45	0.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	59.0
3	46	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0	96.0
3	47	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	96.0
3	48	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	1.0	96.0

(1,1,1), (1,1,2), (1,1,3), ... , (4,4,3). Thus the first cell (1,1,1) is represented by the first row of the matrix as

1.0 0.0 0.0 0.0 1.0 0.0 0.0 0.0 1.0 0.0 0.0 3.0

indicating Level 1 of Factor A, Level 1 of Factor B, and Level 1 of Factor C are present, and the response value is 3.0.

Note that there are 144 rows in this matrix -- three blocks (Column 1) of 48 rows each (Column 2). This is because of the fact that the subject had given three responses to each stimulus combination. NONMETRG, then, can handle replications of stimulus combinations either within or among subjects.

Table 14

History of Computation and Final Configuration from NONMETRG Analysis of the Riskiness Data

ITERATION	THETA	TAU
1	0.84485	-0.35047
2	0.23613	0.59693
3	0.05461	0.80674
4	0.07670	0.76300
5	0.06918	0.80496
6	0.06906	0.76300

	VARIABLE	ADDITIVE	ADDITIVE RESCALED	MULTIP	FROM ITERATION NUMBER 6
1	$L=-.10	0.66792	88.54173	1.95017	
2	$L=-.20	0.20749	42.49893	1.23058	
3	$L=-.30	0.00154	21.90387	1.00154	
4	$L=-.40	-0.21750	0.0	0.80453	
5	PL=1/8	0.61410	83.15974	1.84799	
6	PL=2/8	0.22083	43.83362	1.24712	
7	PL=3/8	0.03066	24.81602	1.03113	
8	PL=4/8	-0.08626	13.12417	0.91736	
9	PW=2/8	0.05189	26.93941	1.05326	
10	PW=3/8	0.16813	38.56279	1.18309	
11	PW=4/8	0.25957	47.70709	1.29637	

Given the appropriate initial parameter values and the data matrix as shown in Table 13, NONMETRG needs one additional matrix of values to begin the scaling procedure. This matrix is actually a vector of n initial scale values to be used by the program as an initial estimate of the stimulus coordinates (i.e., the scale

values for the levels of the factors). Since we are assuming an additive combination rule, the program then generates from this nx1 matrix an nxt matrix of coordinates of each of the n stimuli on the t factors, and finally, an nx1 matrix D′ of scaled distances for each stimulus such that

$$d'_j = x_1 + x_2 + \ldots + x_t \qquad (24)$$

where d'_j is the scaled distance for stimulus j.

To begin the iterative procedure, the program first obtains the corresponding rank orders for the data in each block. After the data have been ranked, every pair of stimuli or factor combinations are compared with respect to differences in their ranks. This difference in rank for each pair i and j (denoted $r_i - r_j$) is compared with the difference in the corresponding distance values (denoted $d'_i - d'_j$) generated from the initial configuration. It should be clear that if we could find a set of scale values, d'_j, that fit an additive model perfectly, then when we compare for every pair of stimuli $(r_i - r_j)$ with $(d'_i - d'_j)$ they would always be of the same sign. In a rank order sense, then, there would be perfect agreement or "fit" to the model. This is exactly what NONMETRG attempts to do. Whe the program finds the two differences to be of the same sign, the distances are treated as being in the right order. If they are not of the same sign, the program must try to modify one or both of the distances so that the signs will be the same. At the same time, the distances must not be changed completely independently of their effects on other comparisons. Thus, the iterative procedure in the program is based on the attempt to change the confiquration of points so as to reduce the number of discrepancies between ranks and

distances as illustrated above for all pairs of stimulus points. To accomplish this, Johnson (1973) suggested the following procedure.

We begin by defining a measure of the goodness-of-fit of the data to the model. The measure is THETA (ϕ) and is defined as

$$\phi = \sqrt{\frac{\sum_{i,j=1}^{n} \left[a \quad * \quad (d_i - d_j) \right]}{\sum_{i,j=1}^{n} \left[(d_i - d_j) \right]}} \qquad (25)$$

where a=0 if sign$(d'_i - d'_j)$ = sign$(\hat{r}_i - \hat{r}_j)$ and a=1 otherwise. THETA would be zero, indicating a perfect fit, only if the signs matched across all n*(n-1)/2 pairs of stimuli. In the worst situation THETA could theoretically equal a maximum of one. What we have is a suitably normalized measure of goodness-of-fit based on the sum of squared differences in the distances among the stimulus combinations. The numerator can be thought of as a sum of squared departures from monotonicity. It ranges in value from zero to the value of the denominator.

Table 14 presents the results of the scaling of the riskiness data to an additive model. Previous research (Nygren, 1979) suggests that a good fit should be expected for the additive model. Looking first at the history of computation, we find that on the first iteration THETA was very poor -- .84485. This is to be expected for the first iteration since the initial stimulus configuration in this example was based on random numbers. One

Table 15

Additive Scale Values for the 48 Gambles in the NONMETRG Analysis of the Riskiness Data

STIMULUS:	LEVELS			ADDITIVE	RESCALED ADDITIVE
1	1	1	1	1.33390	0.0
2	1	1	2	1.45014	11.62338
3	1	1	3	1.54158	20.76768
4	1	2	1	0.94064	-39.32602
5	1	2	2	1.05688	-27.70271
6	1	2	3	1.14832	-18.55840
7	1	3	1	0.75047	-58.34361
8	1	3	2	0.86670	-46.72021
9	1	3	3	0.95814	-37.57591
10	1	4	1	0.63355	-70.03546
11	1	4	2	0.74978	-58.41208
12	1	4	3	0.84123	-49.26778
13	2	1	1	0.87348	-46.04269
14	2	1	2	0.98971	-34.41930
15	2	1	3	1.08115	-25.27504
16	2	2	1	0.48021	-85.36884
17	2	2	2	0.59645	-73.74544
18	2	2	3	0.68789	-64.60114
19	2	3	1	0.29004	-104.38632
20	2	3	2	0.40627	-92.76302
21	2	3	3	0.49772	-83.61871
22	2	4	1	0.17312	-116.07828
23	2	4	2	0.28935	-104.45480
24	2	4	3	0.38080	-95.31058
25	3	1	1	0.66753	-66.63774
26	3	1	2	0.78376	-55.01436
27	3	1	3	0.87520	-45.87006
28	3	2	1	0.27426	-105.96379
29	3	2	2	0.39050	-94.34048
30	3	2	3	0.48194	-85.19618
31	3	3	1	0.08409	-124.98140
32	3	3	2	0.20032	-113.35802
33	3	3	3	0.29177	-104.21371
34	3	4	1	-0.03283	-136.67325
35	3	4	2	0.08340	-125.04987
36	3	4	3	0.17485	-115.90556
37	4	1	1	0.44849	-88.54163
38	4	1	2	0.56472	-76.91824
39	4	1	3	0.65616	-67.77394
40	4	2	1	0.05523	-127.86769
41	4	2	2	0.17146	-116.24431
42	4	2	3	0.26290	-107.10001
43	4	3	1	-0.13495	-146.88530
44	4	3	2	-0.01872	-135.26192
45	4	3	3	0.07273	-126.11760
46	4	4	1	-0.25187	-158.57715
47	4	4	2	-0.13563	-146.95377
48	4	4	3	-0.04419	-137.80946

could, of course, use a good approximation of the configuration obtained via some previous analysis as a means of improving the efficiency of the iterative process. Notice, however, that on Iteration 2 the THETA value is reduced to .23613. The program was able in one iteration to move the $\hat{d}_j$'s considerably closer to monotonicity. This is also reflected in the TAU value printed in the table. TAU is simply a Kendall's Tau coefficient computed between the rank order of the original data with the rank order of the rescaled distances. As THETA decreases TAU will necessarily increase.

There are several other things to notice from the history section of this table. First, the program stopped after Iteration 6. We could let the program continue to iterate much longer. However, there is generally a practical stopping point for any given analysis. As with virtually all non-metric iterative scaling programs, the procedure is stopped in either of two ways. First, the iterative process will stop if a maximum number of iterations is reached. Usually 20 to 30 iterations is set as the default stopping point. In our example, the program converged very quickly, with only six iterations. This is not uncommon for data with a small amount of error. If the program requires more than 30 iterations to converge, it generally means that there is a significant amount of error in the data or that the program is vascillating back and forth between several equally good (or bad) solutions, or both. The second stopping criterion is based on a measure of improvement in fit from one iteration to another. Generally, an efficient iterative algorithm will, except for occasional minor fluctuations in the iterative process, continue to minimize the goodness-of-fit measure. Theoretically, one should be able to continually iterate to find a progressively

better solution. In practice, however, if the THETA values on Iterations n and n+1 differ by a value of .001 or less, there will be no appreciable differences in the final scale values for each. Hence, the program will have found for all practical purposes, the minimum value of THETA, and the iterative process should be stopped. The user can then specify a cutoff criterion for differences in THETA to allow the program to stop after a practical "best" solution has been found. This is what happened on Iteration 6 in the analysis in Table 14.

Notice that Iteration 6 produced neither the lowest THETA nor the highest TAU. These two measures are not perfectly linearly related to one another, since we are dealing with only ordinal relationships here. The program allows the user to specify whether the solution from the iteration with the lowest THETA should be used in the final scaling or whether the solution from the last iteration should be used. Usually there is little practical significance to this option, since the last iteration will generally produce the lowest THETA. Using the last iteration is generally recommended, since it has been the author's experience with moderate-sized data sets (e.g., 4x4x3 or smaller) that a more interpretable solution will be obtained from the last or next to last iteration even if it has a slightly higher THETA value than some previous early iteration.

How does NONMETRG or any other iterative procedure find the "best" solution? Generally, the procedure is conceptually straightforward, although mathematically and computationally it can become quite involved. Essentially, an attempt is made to

Table 16

Comparison of Original Data and Final Scale Values of Stimuli in Block 1 for Riskiness Data

DEPENDENT & PREDICTIONS SORTED BY DEPENDENT. BLOCK NO. 1.

98.000	-0.044
98.000	-0.252
97.000	-0.136
96.000	-0.019
96.000	-0.033
95.000	0.083
94.000	-0.135
92.000	0.084
89.000	0.173
88.000	0.055
61.000	0.200
59.000	0.073
59.000	0.175
52.000	0.290
51.000	0.274
46.000	0.292
39.000	0.263
39.000	0.171
39.000	0.381
38.000	0.289
29.000	0.482
29.000	0.390
29.000	0.498
29.000	0.406
19.000	0.656
19.000	0.565
19.000	0.448
19.000	0.688
19.000	0.480
19.000	0.841
19.000	0.750
19.000	0.634
18.000	0.596
15.000	0.784
15.000	0.668
15.000	0.958
15.000	0.867
14.000	0.875
14.000	0.750
9.000	0.990
9.000	0.873
9.000	1.148
9.000	0.941
8.000	1.057
6.000	1.081
4.000	1.542
4.000	1.450
3.000	1.334

PREDICTIVE CAPABILITY = 4.521286 PERCENT
OR = 95.478714 IP DATA ARE IN REVERSE ORDER.

minimize the goodness-of-fit measure. In the case of NONMETRG this means minimizing THETA. Let W represent the vector containing the $n_1 + n_2 + \ldots + n_t = m$ scale values for the levels of the t factors, let U represent the numerator of Equation 25, and let V represent the denominator of Equation 25. Then we have $\phi = U/V$. We can use a standard mathematical procedure for minimizing O by finding the gradient for the partial derivatives of ϕ with respect to W. That is, we find

$$\frac{\partial O}{\partial W} = \frac{1}{V} * \left(V * \frac{\partial U}{\partial W} - U * \frac{\partial V}{\partial W} \right) \qquad (26)$$

Thus we need to find the partial derivatives of U and V with respect to W, where U and V are as defined in Equation 24 to be functions of $(d'_i - d'_j)$. The procedure is simplified somewhat since the partial of the sum of squared differences is equal to the sum of the partials of each of the terms being summed.

On any given iteration we proceed as follows. Since we have the matrix W (a set of random or user supplied values on Iteration 1) we can easily compute X, the stimulus coordinates. We will normalize X to have a sum of squares equal to a convenient constant value (usually a value of one). We then compute the gradient, G, and normalize it to have a sum of squares of one also. On the first iteration we have W_0 and X_0, and compute ϕ_0 from them. We find G as indicated in Equation 26 and modify W_0, the initial set of scale values by

$$W_1 = W_0 - (O_0 * G_0) \qquad (27)$$

In general, we have

$$W_i = W_{i-1} - (O_{i-1} * G_{i-1}) \qquad (28)$$

In the minimization process we are attempting to locate the point in "configuration space" where G_i will be exactly zero and no change in W will occur. In practice, the iterative procedure will actually stop when G is hopefully very close to zero.

The remaining information presented in Table 14 indicates the final solution for riskiness data based on Iteration 6. The final scale values for the m=11 levels are presented for an additive model and a multiplicative model. Recall that it was shown earlier that in the positive case the multiplicative and additive models are mathematically equivalent. That is, the additive scale values are merely the logarithms of the multiplicative values. Hence, both are presented in the NONMETRG procedure. In addition, the additive values are conveniently rescaled in such a way that cell (1,1,...,1) is set to zero and each new value is multiplied by 100. Finally, the additive scale values for all stimulus combinations are presented both in the original and rescaled forms in the program. The scale values for the riskiness data are presented in Table 15. Notice that the greatest perceived risk is associated with the smallest scale values. That is, the original data and the final scale values are negatively correlated. Since this ordering is somewhat arbitrary, the NONMETRG program has as an option a means of simply reversing the data values so that large observations are associated with large scale values.

Sometimes it is useful to look at the transformation that has been applied to the data to form the stimulus scale values. NONMETRG provides several additional pieces of information that may aid in this analysis. First, the program prints for each

block of data (in our example a block represents one complete replication or set of scores for the subject), each original data value and its corresponding rescaled value from the additive model -- sorted by the data values. Table 16 illustrates this information for the first block (replication) of observations in the risk study. A useful statistic is presented following the presentation of these values. It is a component of the Kendall's Tau value that could be computed between the two sets of scores. It is the proportion of pairs of final scale values that are in the same order as their corresponding data values. In the case of the riskiness data, this value is quite high, 95.48%. Finally, MONMETRG presents a plot of the corresponding data values and additive scale values within each block. This allows the user to obtain a good estimate of the degree of linearity or nonlinearity in the function relating data and rescaled values. An excellent fit of the data to the model would be indicated by a smooth curve (not necessarily linear) running through all of the pairs of points. This plot for Block 1 in the risk study is presented in Figure 1.

MONANOVA

A second program that is capable of scaling data to fit an additive model is Kruskal's (1965) MONANOVA program. As the name of the program suggests, it too is a non-metric procedure that attempts to find a monotonic transformation of the data such that stimulus combinations are obtained by applying an additive combination rule to the levels of the independent factors. This is equivalent to finding no interactions in an analysis of variance of the transformed data. Hence, the program is referred

to as MONANOVA (Monotonic Analysis Of Variance).

Table 17

History of Computation from MONANOVA

Analysis of Riskiness Data

IT	STRESS	SRAT	SRTAVG	CAGRGL	COSAV	ACSAV	GRMAG	GRMULT	STEP
0	0.221	1.2000	1.2000	0.0	0.0	0.067	0.21191	5.5219	1.17017
1	0.200	1.1054	1.1676	-0.734	-0.490	0.512	0.24620	2.6406	0.65014
2	0.124	1.6156	1.3011	-0.961	-0.804	0.811	0.08954	3.0046	0.26905
3	0.126	0.9806	1.1841	-0.993	-0.930	0.932	0.10638	0.7722	0.08215
4	0.120	1.0540	1.1390	0.999	0.356	0.977	0.04940	1.5106	0.07462
5	0.118	1.0133	1.0955	-0.983	-0.537	0.981	0.00775	2.3781	0.01843
6	0.118	1.0001	1.0627	-0.997	-0.844	0.992	0.00659	0.4307	0.00284
7	0.118	1.0001	1.0414	1.000	0.385	0.997	0.00438	0.4903	0.00215
8	0.118	1.0001	1.0274	1.000	0.795	0.999	0.00270	1.0275	0.00278
9	0.118	1.0000	1.0182	0.996	0.929	0.997	0.00054	8.2718	0.00448
10	0.118	1.0000	1.0121	-0.999	-0.357	0.999	0.00293	0.3847	0.00113
11	0.118	1.0000	1.0081	1.000	0.548	1.000	0.00206	0.4792	0.00099
12	0.118	1.0000	1.0054	1.000	0.849	1.000	0.00129	1.0057	0.00130
13	0.118	1.0000	1.0036	0.996	0.947	0.997	0.00028	7.3671	0.00210
14	0.118	1.0000	1.0024	-0.999	-0.350	0.999	0.00134	0.3896	0.00052
15	0.118	1.0000	1.0016	1.000	0.550	0.999	0.00093	0.4811	0.00045
16	0.118	1.0000	1.0011	1.000	0.850	1.000	0.00059	1.0006	0.00059
17	0.118	1.0000	1.0007	0.996	0.947	0.997	0.00013	7.1704	0.00095
IT	STRESS	SRAT	SRTAVG	CAGRGL	COSAV	ACSAV	GRMAG	GRMULT	STEP

APPROXIMATE MINIMUM WAS REACHED. FINAL CONFIGURATION HAS STRESS OF 0.1180

Conceptually, NONMETRG and MONANOVA are quite similar in that both seek to find scale values for the factor levels with respect to an additive model. The specific algorithms associated with the two programs are quite different, however, since MONANOVA minimizes a different goodness-of-fit measure. The MONANOVA algorithm is based on Kruskal's earlier extensions of Shepard's original non-metric multidimensional scaling procedure for the analysis of similarities data (cf. Kruskal, 1964a, 1964b; Shepard, 1962a, 1962b). Kruskal showed how it was possible to obtain a matrix of interval-scaled distances that was a least squares

estimate to a matrix of monotonically transformed data values. That is, the procedure finds and applies a monotonic transformation to the original data such that a set of distances can be estimated for the stimuli with the constraint that the distances "best" fit the monotonically transformed data in a least squares sense. This suggests a new goodness-of-fit measure which Kruskal (1965) called STRESS>. To define this measure, however, we must first describe the monotonically transformed data matrix which we will call disparities, and will denote as $\hat{d}_{ij}$. If we let S represent our original data matrix, $\hat{D}$ be the disparities matrix, D' be the distances matrix, and X be the final configuration then the procedure can be illustrated as

$$S \overset{m}{=} \hat{D} \doteq D' = g(X) \tag{30}$$

Here $S \overset{m}{=} \hat{D}$ indicates that a monotonic transformation is applied to the data in such a way that a set of disparities are formed, subject to the constraint that

$$\begin{aligned} &\text{(i)} \quad \text{if } s_i > s_j, \text{ then } d'_i \geq d'_j \text{ and} \\ &\text{(ii)} \quad \text{if } s_i = s_j, \text{ then } d'_i = d'_j. \end{aligned} \tag{31}$$

Equation 31 is often referred to as weak monotonicity. It is an important concept and its practical significance will be discussed later in the CPSCAL manual. The portion of Equation 30 written as $\hat{D} \doteq D'$ indicates that a least squares procedure is used to fit distances to the disparity values, subject to the constraint that the d'_{ij}'s satisfy the standard distance properties and $D' = g(X)$ where $g(X)$ is defined in the case of the additive model to be

$$d_j = f_1(x_{j1}) + f_2(x_{j2}) + \ldots + f_t(x_{jt}) \qquad (32)$$

and x is the level of Stimulus j on Factor i.

Table 18

Final Configuration and Stimulus Scale Values for MONANOVA Analysis of Riskiness Data

```
ACB DESIGN.  4X4X3 DESIGN.  $L, PL, PW.
  1  -1.7999   -0.1881      0.6659   1.3222
  2  -1.8258   -0.1436      0.6668   1.3027
  3   0.1477   -0.0245     -0.1232

ADDITIVE SCALE VALUES FOR    46 STIMULI.
STIMULUS:  LEVELS          ADDITIVE      RESCALED
    1    1   1   1          -3.47809        0.0
    2    1   1   2          -3.65023     -17.21362
    3    1   1   3          -3.74894     -27.08511
    4    1   2   1          -1.79589     168.21965
    5    1   2   2          -1.96803     151.00603
    6    1   2   3          -2.06674     141.13454

                .
                .
                .

   43    4   3   1           2.13659     561.46777
   44    4   3   2           1.96445     544.25415
   45    4   3   3           1.86574     534.38257
   46    4   4   1           2.77251     625.05981
   47    4   4   2           2.60037     607.84619
   48    4   4   3           2.50166     597.97461
PREDICTIVE CAPABILITY = 90.763397 PERCENT
OR =  9.236603 IF DATA ARE IN REVERSE ORDER.
```

As with NONMETRG we need several pieces of information to begin the MONANOVA program. First, we need the parameter values that describe the input data and the independent factors. Next, we need the original data values in the matrix S. Finally, we need a starting configuration, X, consisting of an initial set of scale values for the levels of the factors. This last matrix is

important since a "good" starting configuration may lead to convergence to an optimal solution very quickly, reducing computer time as well as possibly reducing the likelihood of the program stopping at a non optional or "local minimum" solution.

In MONANOVA, this initial configuration can either be supplied by the user or be created in the program. When generated by the program the initial configuration is created in a rather arbitrary way and generally produces a poor fit of the model to the data. Regardless of how this initial configuration is generated, it is then normalized. By this we mean that the mean of the scale values of the m levels of the factors is 0.0 and the sum of squares of the scale values is m. This normalization procedure will in no way hinder the iterative procedure, since as we shall see when the STRESS measure is defined, it is totally invariant to both translation and uniform stretching of the stimulus space.

Once the initial configuration, X, has been generated, it is easy to compute the d'_j's in the distance matrix, D', by using the additive composition rule. We now have the ingredients for Kruskal's goodness-of-fit measure. If a monotonic transformation of the data (i.e., the S matrix) could be found such that when the s_j's are transformed into the disparities, $\hat{d}_j$, these $\hat{d}_j$'s could be treated exactly as distances, d'_j, satisfying the additive model perfectly, then we would have $\hat{d}_j = d'_j$ for every stimulus j. This suggests that a measure of goodness-of-fit might be the sum across all stimuli of a function of the discrepancies between the disparity and distance values. This is exactly what STRESS is.

Table 19

History of Computation from DISTRIB

Analysis of Riskiness Data: A*(B+C)

IT	STRESS	SRAT	SRTAVG	CAGRGL	COSAV	ACSAV	GRMAG	GRMULT	STEP
0	0.3465	1.2000	1.2000	0.0	0.0	0.067	0.1450	8.6618	1.2565
1	0.1748	1.9819	1.4184	0.848	0.565	0.588	0.0938	39.9433	3.7489
2	0.6962	0.2511	0.7965	-0.726	-0.295	0.680	0.1673	5.1330	0.8589
3	0.5426	1.2830	0.9336	0.977	0.553	0.878	0.1876	5.6952	1.0684
4	0.3471	1.5633	1.1087	0.962	0.825	0.934	0.1724	15.7342	2.7132
5	0.2570	1.3504	1.1840	-0.810	-0.265	0.851	0.1107	12.5933	1.3949
6	0.1727	1.4885	1.2779	-0.730	-0.575	0.771	0.0467	15.1527	0.7082
7	0.1785	0.9676	1.1647	-0.812	-0.733	0.798	0.0745	3.6369	0.2711
8	0.1640	1.0879	1.1385	0.925	0.372	0.883	0.0313	9.9091	0.3104
9	0.1625	1.0092	1.0937	-0.713	-0.351	0.770	0.0331	4.4277	0.1468
10	0.1598	1.0170	1.0675	0.309	0.089	0.463	0.0117	12.3183	0.1444
11	0.1583	1.0094	1.0478	0.718	0.508	0.633	0.0122	21.0217	0.2566
12	0.1574	1.0060	1.0357	-0.182	0.048	0.332	0.0305	7.5817	0.2315
13	0.1572	1.0011	1.0227	-0.820	-0.531	0.657	0.0354	2.5933	0.0919
14	0.1551	1.0139	1.0197	0.830	0.377	0.773	0.0134	7.6772	0.1034
15	0.1544	1.0044	1.0146	-0.035	0.102	0.281	0.0155	6.3229	0.0980
16	0.1538	1.0041	1.0111	-0.187	-0.090	0.218	0.0154	4.6057	0.0709
17	0.1531	1.0040	1.0087	0.153	0.071	0.174	0.0113	5.6230	0.0639
18	0.1526	1.0036	1.0070	0.524	0.373	0.408	0.0109	7.9875	0.0872
19	0.1519	1.0044	1.0061	0.336	0.348	0.360	0.0129	8.8537	0.1148
20	0.1515	1.0028	1.0050	-0.280	-0.071	0.307	0.0207	4.0009	0.0830
21	0.1507	1.0050	1.0050	-0.238	-0.182	0.261	0.0122	4.2640	0.0521
22	0.1503	1.0029	1.0043	0.414	0.215	0.363	0.0102	5.4874	0.0564
23	0.1498	1.0031	1.0039	0.6[illegible]	0.483	0.533	0.0103	8.6424	0.0890
24	0.1492	1.0041	1.0040	0.2[illegible]	0.319	0.335	0.0140	7.9926	0.1120
25	0.1491	1.0009	1.0029	-0.486	-0.217	0.436	0.0246	2.6493	0.0653
26	0.1482	1.0059	1.0039	0.219	0.073	0.291	0.0090	6.3729	0.0576
27	0.1477	1.0034	1.0038	0.932	0.646	0.719	0.0091	12.4162	0.1130
28	0.1468	1.0060	1.0045	0.520	0.562	0.586	0.0124	16.0709	0.1993
29	0.1489	0.9860	0.9983	-0.619	-0.226	0.608	0.0550	1.8776	0.1033
30	0.1455	1.0233	1.0066	0.811	0.466	0.744	0.0123	11.3779	0.1404
IT	STRESS	SRAT	SRTAVG	CAGRGL	COSAV	ACSAV	GRMAG	GRMULT	STEP

MAXIMUM NUMBER OF ITERATIONS WAS USED.
FINAL CONFIGURATION HAS STRESS = 0.14552

Formally, we define it as

$$S = \frac{\sum_{j=1}^{n} (\hat{d}_j - d'_j)^2}{\sum_{j=1}^{n} (d'_j - \bar{d})^2} \qquad (33)$$

where n is the number of stimulus combinations, d'_j and $\hat{d}_j$ are

defined as before, and $\bar{d}$ is the mean distance value. Originally, Kruskal had suggested that STRESS be defined by simply using the sum of squares of the distances in the denominator. However, he found that in some situations the likelihood of degenerate or non-optimal solutions could be avoided if the denominator were actually the sum of squared deviations from the mean of the distances. Other measures of goodness-of-fit related to STRESS have been proposed, including one in which the mean distance, $\bar{d}$, in the denominator of Equation 33 is replaced by $\bar{\hat{d}}$, the mean disparity value. Regardless of how STRESS is defined, it is clear that a perfect solution will be found when the original data can be monotonically transformed into disparities that fit the distance model perfectly. This leads to a STRESS of zero. The goal of the MONANOVA program is then to find distances that will minimize STRESS. To accomplish the minimization process, MONANOVA uses basically the same means that was used in NONMETRG. That is, the numerical method of gradients or steepest descent is used to find the minimal value of STRESS (or more accurately, the minimal value of the square of the STRESS value, S^2).

Before the minimization process can be done, however, a method for computing the disparities must be constructed. There are several methods for doing this, although Kruskal's (1964) monotonicity principle is most often used. In the monotonic regression procedure devised by Kruskal we attempt to find values of $\hat{d}'_j$ from the given values of δ_j under the constraints of montonicity as defined in Equation 31. On the first iteration the given values of the d'_j's come from the user-supplied or program-generated initial configuration. The procedure to be described

Table 20

Final Configuration and Stimulus Scale Values for DISTRIB Analysis of Riskiness Data: A*(B+C)

```
FINAL ITERATION CONFIGURATION:
  1    2.1563    3.7007    4.6566    5.5832
  2    0.2069    2.8841    4.4955    6.0103
  3    3.8372    3.3831    3.0611

FINAL STRESS AND MINIMUM STRESS ARE EQUAL.
THE PREDICTED DISTRIBUTIVE SCALE VALUES FOR  48 STIMULI:
FINAL ITERATION CONFIGURATION:
 SEQ. NO.       DATA     SUBSCRIPTS

    1        8.72031     1     1     1
    2        7.74130     1     1     2
    3        7.04681     1     1     3
    4       14.49306     1     2     1
    5       13.51405     1     2     2
    6       12.81956     1     2     3

    .
    .
    .

   43       46.52243     4     3     1
   44       43.98755     4     3     2
   45       42.18938     4     3     3
   46       54.97975     4     4     1
   47       52.44485     4     4     2
   48       50.64668     4     4     3

PRECAP VALUE FOR FINAL ITERATION CONFIGURATION.
PREDICTIVE CAPABILITY = 90.637131 PERCENT
```

below, although it is not identical to Kruskal's monotone regression procedure as actually programmed in MONANOVA, it is very similar to this method, it is easier to understand intuitively, and produces final disparity values that are identical to those obtained from the actual method. For these reasons, this modified procedure devised by Young (1973) will be described.

We begin by ordering the data values from smallest to

largest. From this point on we will need only this rank ordering of the data and not the data values themselves. Again, this is because the procedure is nonmetric. We begin by assigning the first and second disparities, $\hat{d}_1$ and $\hat{d}_2$, the values of the distances from the initial configuration that correspond to our smallest and second smallest data values. That is, $\hat{d}_1 = d'_1$ and $\hat{d}_2 = d'_2$. If $\hat{d}_1 < \hat{d}_2$, then they are in the correct order and are what Kruskal referred to as being down-satisfied. If this ordering is correct, then we go on and set $\hat{d}_3 = d_3$. If, however, $\hat{d}_2 > \hat{d}_1$, then each is replaced by $\hat{d}_{12}$, the average of the two. In this way, the "new" $\hat{d}_1$ and $\hat{d}_2$ will be weakly monotonic with the original data, and we will have a block of two equal disparities. The process continues with $\hat{d}_3$ being set equal to d'_3. It is then compared with $\hat{d}_2$ or with the block consisting of $\hat{d}_1$ and $\hat{d}_2$ if they are equal. If the ordering here is correct ($\hat{d}_2 < \hat{d}_3$) then we go on to $\hat{d}_4$. If not, then $\hat{d}_2$ and $\hat{d}_3$ (or if $\hat{d}_1 = \hat{d}_2$, $\hat{d}_1$, $\hat{d}_2$, and $\hat{d}_3$) are set equal to their average, $\hat{d}_{123}$ (or $\hat{d}_{123}$). The process is continued until all observations have been ordered. In this procedure we are always making comparisons in the down-satisfied direction. In Kruskal's procedure there is actually an alternating process whereby each block is compared with the next smaller block as well as the next larger block. Thus, a particular block is also being made up-satisfied. However, when we have completed the process described above, all blocks will, in fact, be both up-and down-satisfied, yielding final disparities that are identical to those that would be obtained using Kruskal's method.

Given the disparities and distances, STRESS can now be computed for Iteration 1. Since our initial configuration may be very poor, we might expect large discrepancies between the $\hat{d}_j$'s

and the d_j's. This would produce a high initial STRESS value. We next compute the partial derivatives (i.e., the gradients) of S with respect to the configuration coordinates, X. The negative gradient is then calculated. This allows us to move the configuration in a direction where S will be decreasing most rapidly (i.e., steepest descent). In the MONANOVA program this allows us to compute a parameter called the step size which then leads to the creation of a new configuration. This new configuration should be much better than the initial one used on Iteration 1.

Table 21

Reanalysis of Riskiness Data for New DISTRIB Model with Factor C as the Outside Factor

DISTRIBUTIVE MODEL ANALYSIS FOR C*(B+A) DESIGN.

SEQ. NO.	DATA	SUBSCRIPTS C	B	A
1	3.00000	1	1	1
2	5.00000	1	1	1
3	5.00000	1	1	1
4	9.00000	1	1	2
5	9.00000	1	1	2
6	9.00000	1	1	2
.				
.				
.				
139	59.00000	3	4	3
140	59.00000	3	4	3
141	59.00000	3	4	3
142	98.00000	3	4	4
143	97.00000	3	4	4
144	96.00000	3	4	4

We can now begin Iteration 2, repetition of Iteration 1 beginning with the normalization of the new configuration found on the last iteration. As the iterative process continues, the

STRESS value should continue to decrease until there is no practical improvement in this fit measure and, consequently, in the final configuration. As with NONMETRG there needs to be a stopping criterion in MONANOVA. There are two such criteria that are directly comparable to those found in NONMETRG. First, the user may specify a maximum number of iterations. Usually, 30 is sufficient to insure convergence. Secondly, there is an improvement criterion associated with the change in STRESS from Iteration n to Iteration n+1. Usually this criterion is set at a value between .001 and .0001.

Table 17 presents the history of computation from the MONANOVA analysis of the riskiness data used in the previous analyses. This history provides several useful pieces of information not directly related to the final scaling solution. The columns represent the values on each iteration of the parameters used in the iterative process. Aside from the STRESS value, the other parameter that might be of value is the STEP size. The STEP size indicates the "movement" in the configuration values from one iteration to the next. Note that in our example the STRESS value had stabilized and the STEP size was very close to zero. This indicates little improvement in the last eleven iterations. As it does in most cases with low or moderately low amounts of error in the data, the MONANOVA program converged very rapidly to a final additive configuration.

The final configuration and the scale values for 12 of the 48 stimuli are presented in Table 18. As was mentioned in the discussion of the normalization process, the mean of the scale values in the configuration is zero and the sum of squares of these values adds to eleven. This normalization is arbitrary, of

course, since the scales are interval. It is important to recognize, however, that the scale values for the levels of the different factors can be compared. In this example, the scale values for Amount to Lose ($L) ranged from -1.7999 to 1.3222. For Probability of Losing (PL) the range was about the same, being -1.8258 to 1.3027. For the Probability of Winning factor (PW), the range was only 0.1477 to -0.1232. These scale values are in agreement with what was suggested earlier in the CONJOINT and PCJM2 analyses of this subject's riskiness ratings. Both Amount to Lose and Probability of Losing were very important in determining risk for this subject. In fact, they appear to be about equally weighted because their ranges are comparable. Probability of Winning had virtually no effect on perceived risk. In this example, the levels of Probability of Winning were 2/8, 3/8, and 4/8. For this subject, then, the MONANOVA results indicate that perceived risk would be essentially unaffected if the Probability of Winning were doubled from 2/8 to 4/8, assuming that the other factors were held constant.

A final piece of useful information provided by the MONANOVA program is a plot of the monotonic transformation applied to the data values. Just as with NONMETRG, this plot should indicate a nice, smooth curve if the data are being fit perfectly by the model. The plot for the riskiness data is found in Figure 2. This plot is monotonic as was expected from the results in Tables 17 and 18. The STRESS value was low (0.1180) and the Predictive Capability value was high (90.76%). The Predictive Capability measure is interpreted in the same way as it was with NONMETRG. It is the percentage of data values and final distances that is in the right order in a pairwise comparison sense. In the case of the risk analysis 90.76% of the (48*47)/2 = 1128

comparisons were in the same order for the original data and the final distances.

V. ADDITIONAL CONJOINT SCALING PROGRAMS: DISTRIB AND DUALDIST

There are a number of other programs that will also do additive conjoint scaling (cf. Carroll's (1972) MDPREF, Young's (1972, 1980) POLYCON and ADDALS, and Srinivasan and Shocker's (1973) LINMAP. There are, however, two other programs that are currently available for doing a conjoint analysis via a distributive or dual-distributive model. Both of these programs were adapted from MONANOVA by Emery (1977a, 1977b). These programs were called DIST and DULDIST respectively. They have been further revised in a manner to be described below into the DISTRIB and DUALDIST programs in the CPSCAL manual.

There is no need to describe in detail the non-metric iterative procedure employed by the DISTRIB and DUALDIST programs since they are virtually identical to that of MONANOVA. In fact, Emery's original DIST and DULDIST Fortran programs are identical to the MONANOVA program except for about ten of the 1000 program statements. All that is changed in these two programs are statements allowing for the computation of the distances from a distributive or dual-distributive model rather than an additive model, and appropriate modifications to the minimization subroutine.

The primary change made to the DIST and DULDIST programs by the author for the CPSCAL manual was an extension to allow for the testing of more than one form of the models. In an additive

Table 22

Final Configuration and Stimulus Scale Values from New DISTRIB Analysis of Riskiness Data: C*(B+A)

```
FINAL ITERATION CONFIGURATION:
  1  17.5823   15.9346   14.4515
  2  -8.7296    1.2336    6.2670    9.9324
  3  -8.5860    1.0484    6.2461    9.9949

MAXIMUM NUMBER OF ITERATIONS WAS USED.
FINAL CONFIGURATION HAS STRESS = 0.11031


THE PREDICTED DISTRIBUTIVE SCALE VALUES FOR  48 STIMULI:
     SEQ. NO.    DATA          SUBSCRIPTS
                                C     B     A
         1     -304.44604       1     1     1
         2     -135.05205       1     1     2
         3      -43.66588       1     1     3
         4       22.24666       1     1     4
         5     -129.27139       1     2     1
         6       40.12306       1     2     2

         .
         .
         .

        43      180.83215       3     3     3
        44      235.00815       3     3     4
        45       19.45753       3     4     1
        46      158.68925       3     4     2
        47      233.80283       3     4     3
        48      287.97876       3     4     4

    PRECAP VALUE FOR FINAL ITERATION CONFIGURATION.
    PREDICTIVE CAPABILITY = 90.345764 PERCENT
```

analysis it makes no difference whether the factors are read as A+B+C, B+C+A, or C+A+B; the results will be identical. For a distributive model, however, one may want to investigate the fit of all three distinct forms of the model: A*(B+C), B*(A+C), and C*(A+B). Each of these would produce very different scaling solutions, since in each case there is a different multiplicative factor. In a comparable manner we can define three distinct dual-distributive models: A+(B*C), B+(A*C), and C+(A*B). DISTRIB and DUALDIST allow the user to obtain a scaling solution for all three

Table 23

Final Configuration and Stimulus Scale Values for DUALDIST Analysis of Riskiness Data: A+(B*C)

```
FINAL ITERATION CONFIGURATION.
  1   -1.2534    2.8208    5.0111    6.6830
  2    1.3106    2.9658    3.7756    4.4077
  3    2.6976    2.5455    2.4608

APPROXIMATE MINIMUM WAS REACHED.
MAXIMUM NUMBER OF ITERATIONS WAS USED.
FINAL CONFIGURATION HAS STRESS = 0.11354
THE PREDICTED DUAL-DISTRIBUTIVE SCALE VALUES
FOR  48 STIMULI:
     SEQ. NO.    DATA          SUBSCRIPTS
                                A     B     C
         1        2.28201       1     1     1
         2        2.08268       1     1     2
         3        1.97169       1     1     3
         4        6.74721       1     2     1
         5        6.29612       1     2     2
         6        6.04496       1     2     3

         .
         .
         .

        43       16.86804       4     3     1
        44       16.29378       4     3     2
        45       15.97406       4     3     3
        46       18.57320       4     4     1
        47       17.90279       4     4     2
        48       17.52953       4     4     3

PRECAP VALUE FOR FINAL ITERATION CONFIGURATION.
PREDICTIVE CAPABILITY = 90.773102 PERCENT
```

forms of the models in one analysis. This is not only potentially interesting in terms of the data, but it is also convenient, since the data need only be stored in one form.

Tables 19 and 20 present the results of the DISTRIB analysis of the riskiness data assuming a model of the form A*(B+C) or specifically, $L*(PL+PW). Again, since the programs are very simple extensions of MONANOVA, the values in Tables 19 and 20 are interpreted in the same manner as were Tables 17 and 18. Tables 21 and 22 show the results of the reanalysis of the risk data to

fit the distributive model C*(B+A). Table 21 merely indicates how the data have been internally rearranged to be in the appropriate sequence for the C*(B+A) model. Table 22 shows the final configuration and final scale values for 12 of the 48 stimuli for this model. Finally, Table 23 illustrates the final solution for the data as it was fit by DUALDIST according to the A+(B*C) model.

It should be apparent from looking at the STRESS values and the Predictive Capability values from the MONANOVA analysis, the two DISTRIB analyses, and the DUALDIST analysis of the same riskiness data that there are only small differences in the fit of the models. This seems quite surprising since the models are conceptually and mathematically very different. However, recent research by Emery and Barron (1979) and others suggests that the goodness-of-fit measures like STRESS may not be very powerful in many situations in differentiating among the four models discussed in this manual. Clearly, a great deal of research needs to be done in order for investigators to make the greatest use of these scaling techniques.

APPENDIX 1

THIS APPENDIX PROVIDES STEP-BY-STEP INSTRUCTIONS FOR SETTING UP AND RUNNING SIX DIFFERENT CONJOINT TESTING AND SCALING PROGRAMS. IN EACH OF THE DESCRIPTIONS OF THE PROGRAMS THERE IS A SMALL SECTION SET OFF BY *'S. THIS SECTION DESCRIBES THE NECESSARY JCL CARDS FOR RUNNING THE SOURCE DECK VERSION OF THE PROGRAMS. IN EACH CASE CARDS 1-4 ARE UNIQUE TO THE AMDAHL 470 SYSTEM AT OHIO STATE UNIVERSITY. USERS OF CPSCAL WILL HAVE TO MODIFY THESE CARDS TO FIT THE REQUIREMENTS OF THEIR OWN INSTALLATIONS.

AN EFFORT HAS BEEN MADE TO FIND ALL TYPOGRAPHICAL ERRORS AND INACCURACIES IN THIS MANUAL. NEVERTHELESS, SOME MINOR PROBLEMS MAY STILL EXIST. IF INACCURACIES ARE FOUND PLEASE REPORT THEM TO THE AUTHOR.

C O N J O I N T

```
**********************************************************
*                                                        *
*  C   O   N   J   O   I   N   T                         *
*                                                        *
*  INSTRUCTIONS FOR USING CONJOINT.OPT2                  *
*                                                        *
**********************************************************
**********************************************************
*                                                        *
*  CONJOINT IS A COMPUTER PROGRAM FOR EVALUATING         *
*  CERTAIN CONJOINT-MEASUREMENT AXIOMS.  IT WAS          *
*  WRITTEN BY JAMES O. HOLT III AND THOMAS S.            *
*  WALLSTEN.                                             *
*                                                        *
*  THE PROGRAM HAS BEEN MODIFIED AND ADAPTED FOR         *
*  USE ON THE AMDAHL 470 OPERATING SYSTEM AT             *
*  THE OHIO STATE UNIVERSITY BY:                         *
*  DR. THOMAS E. NYGREN                                  *
*  DEPT. OF PSYCHOLOGY                                   *
*  OHIO STATE UNIVERSITY                                 *
*                                                        *
**********************************************************

************************************************************
*                                                          *
*  TO CALL THIS PROGRAM FOR THE SOURCE DECK FORM FROM      *
*  THE TAPE "CPSCAL", USE THE FOLLOWING JCL CARDS:         *
*                                                          *
*  1)  YOUR ID CARD                                        *
*  2)  //  TIME=2,REGION=300K                              *
*  3)  /*JOBPARM LINES=10000,DISKIO=400,TAPEIO=400        *
*  4)  /*SETUP UNIT=TAPE9,ID=(CPSCAL,XXXX,READ)          *
*  5)  //  EXEC PLIXG,TIME.GO=2,REGION.GO=300K           *
*  6)  //GO.SYSLIN DD UNIT=3400,VOL=SER=CPSCAL,          *
*  7)  //  DISP=(OLD,PASS),DSN=CONJOINT,                 *
*  8)  //  LABEL=(1,SL),                                 *
*  9)  //  DCB=(RECFM=FB,LRECL=80,BLKSIZE=1600)          *
* 10)  //GO.SYSIN DD *                                   *
*                                                        *
* 11)  INPUT DECK AS DESCRIBED BELOW ( CONTROL AND       *
*      DATA CARDS GO HERE.)                              *
*                                                        *
* 12)  /*                                                *
* 13)  //                                               *
*                                                        *
************************************************************
```

****

*

* I N P U T D E C K A R R A N G E M E N T *

*

****

CARD 1. THE "TASK" CARD.

THIS CARD IS USED TO SET UP THE PARAMETERS OF CONJOINT. "TASK" MUST BE PUNCHED IN COLUMNS 1-4. COLUMN 5 IS BLANK. THEN, ALL OF THE FOLLOWING MUST APPEAR IN ORDER AND SEPERATED BY ONE OR MORE SPACES:

NUMBER OF DATA SETS TO BE ANALYZED (UNLIMITED) .
NUMBER OF LEVELS OF FACTOR A (FROM 1 TO 6) .
NUMBER OF LEVELS OF FACTOR B (FROM 1 TO 6) .
NUMBER OF LEVELS OF FACTOR C (FROM 1 TO 6) .
READIN METHOD.
= 0, IF MORE THAN ONE OBSERVATION PER CELL AND DATA CELLS ARE TO BE RANK ORDERED USING THE MANN-WHITNEY U-TEST PROCEDURE.
= 1, IF THERE IS ONE OBSERVATION PER CELL AJD NO RANKING IS REQUIRED.
= 2, IF MORE THAN ONE OBSERVATION PER CELL AS IN OPTION "0" AND THE DATA CELLS ARE TO BE RANK ORDERED. THIS OPTION DIFFERS FROM OPTION "0" ABOVE IN THAT THE DATA ARE ASSUMED TO BE IN A PRESPECIFIED ORDER FOR READIN PURPOSES. THIS SPECIAL READIN ORDER IS DESCRIBED BELOW.
DISTRIBUTIVE CANCELLATION PRINTING OPTION.
= 0, IF THE LISTING OF THE SUBMATRICES VIOLATING DISTRIBUTIVE CANCELLATION IS TO BE SUPPRESSED.
= 1, IF THE LISTING IS TO BE PROVIDED.
DOUBLE CANCELLATION PRINTING OPTION.
= 0, IF THE LISTING OF THE SUBMATRICES VIOLATING DOUBLE CANCELLATION IS TO BE SUPPRESSED.
= 1, IF THE LISTING IS TO BE PROVIDED.

CARD 2. THE "TITLE" CARD.

THIS IS AN OPTIONAL CARD AND IS USED TO PROVIDE A TITLE FOR THE OUTPUT. "TITL" MUST BE PUNCHED IN COLUMNS 1-4. COLUMN 5 IS BLANK. ANY STRING UP TO 73 CHARACTERS, AND ENCLOSED BY SINGLE QUOTES, CAN BE USED.

CARD(S) 3. THE "TEST" CARDS.

ONE TEST CARD IS NECESSARY FOR EACH OF THE SIX TESTS DESIRED. THE TESTS ARE IDENTIFIED BY THE NUMBERS 1 - 6. IN EACH CASE THE WORD "TEST" APPEARS IN COLUMNS 1-4, AND COLUMN IS BLANK. THEN THE FOLLOWING MUST APPEAR IN ORDER AND SEPARATED BY ONE OR MORE SPACES:

TEST NUMBER. (ONE OF THE FOLLOWING MUST APPEAR.)
= 1, IF DOUBLE CANCELLATION IS TO BE TESTED.
= 2, IF SIMPLE INDEPENDENCE IS TO BE TESTED.
= 3, IF SIGN DEPENDENCE IS TO BE TESTED.
= 4, IF DISTRIBUTIVE CANCELLATION IS TO BE TESTED.
= 5, IF JOINT INDEPENDENCE IS TO BE TESTED.
= 6, IF JOINT SIGN DEPENDENCE IS TO BE TESTED.

NUMBER OF OUTSIDE FACTORS.
= 1, 2, OR 3. IN EACH OF THE TESTS ABOVE ONE FACTOR SERVES AS THE "OUTSIDE FACTOR" EACH TIME. FOR EXAMPLE, WHEN CHECKING DOUBLE CANCELLATION ONE MIGHT LOOK AT FACTORS A AND B, A AND C, OR B AND C. THE OUTSIDE FACTOR IN EACH OF THESE CASES WOULD BE FACTORS C, B, AND A RESPECTIVELY. HENCE, IF ONE WANTED TO CHECK DOUBLE CANCELLATION IN ALL THREE OF THESE CASES, ONE WOULD INDICATE THAT THERE ARE "3" OUTSIDE FACTORS. THE NEXT PARAMETER INDICATES WHICH OF THE FACTORS ARE TO BE CONSIDERED THE OUTSIDE FACTOR(S).

DESIGNATION OF OUTSIDE FACTOR(S).
THIS PARAMETER INDICATES WHICH FACTORS ARE TO BE CONSIDERED THE OUTSIDE FACTOR(S). THE LETTERS A, B, AND C, PLACED IN SINGLE QUOTES AND SEPARATED BY ONE OR MORE SPACES ARE USED (E.G., 'A' 'B' 'C'). THE NUMBER OF LETTERS PLACED IN QUOTES MUST MATCH THE NUMBER OF OUTSIDE FACTORS AS INDICATED BY THE PARAMETER ABOVE.

CARD 4. THE LABEL CARD FOR FACTOR A - "ALBL".

THIS CARD PROVIDES A LABEL ON THE COMPUTER PRINTOUT OUTPUT FOR FACTOR A. THE WORD "ALBL" APPEARS IN COLUMNS 1-4, AND COLUMN 5 IS BLANK. COLUMNS 6 - 8 CONTAIN 'A'. ANY STRING OF UP TO 25 CHARACTERS MAY BE USED FOR THE LABEL. THIS STRING MUST BE ENCLOSED IN SINGLE QUOTES, BEGINNING IN COLUMN 10.

CARD(S) 5. LABEL CARDS FOR FACTORS B AND C.

THESE CARDS CORRESPOND TO THE "ALBL" CARD ABOVE, EXCEPT THAT "BLBL" AND "CLBL" APPEAR IN COLUMNS 1 - 4, RESPECTIVELY.

CARD(S) 6. THE DATA CARDS.

THE DATA CARDS ARE SET UP DIFFERENTLY DEPENDING ON THE READIN METHOD INDICATED ON CARD 1.

IF READIN = 0, THE FORMAT IS:

COLUMN	MEANING
1 - 2	DATA SET ID, FROM 01 TO 98.
4	INDEX ON FACTOR A.
6	INDEX ON FACTOR B.
8	INDEX ON FACTOR C.
33 - 41	DATA VALUE IN F9.5 FORMAT.
45	INCLUSION FLAG. = BLANK OR 0 OR 1, IF THE CARD IS TO BE INCLUDED IN THE DATA SET. = 2 OR 3, IF THE CARD IS NOT TO BE INCLUDED.

READIN = 0 IMPLIES THAT THERE ARE 3 OR MORE REPLICATIONS OF EACH JUDGMENT IN EACH DATA SET. THE ENTIRE CARD IS SKIPPED IF COLUMNS 1 AND 2 ARE EMPTY. THE NUMBER OF CARDS, OR DATA ENTRIES PER CELL CANNOT BE LESS THAN 3 NOR EXCEED 50, AND THE ENTIRE NUMBER OF CARDS PER DATA SET CANNOT EXCEED 2000. IT IS NOT NECESSARY TO INCLUDE BLANK CARDS FOR EMPTY CELLS, NOR IS IT NECESSARY TO ARRANGE THE CARDS IN A PARTICULAR ORDER.
EACH DATA SET MUST BE FOLLOWED BY A CARD WITH "99" IN COLUMNS 1 AND 2. THIS SIGNALS THE END OF THE DATA SET AND INITIATES THE ANALYSIS. THE NUMBER OF DISTINCT DATA SETS MUST EQUAL THE NUMBER INDICATED ON THE TASK CARD.

IF READIN = 1, THE FORMAT IS:

COLUMN	MEANING
1 - 2	DATA SET ID, FROM 01 TO 98.
4	INDEX ON A VARIABLE.
6	INDEX ON B VARIABLE.
8	INDEX ON C VARIABLE.
10 - 11	THE NUMBER OF OBSERVATIONS ON WHICH THE CELL ENTRY (DATUM) IS BASED. (IF THIS IS LEFT BLANK, THE CELL WILL BE TREATED AS EMPTY.)
33 - 41	THE DATA VALUE IN F9.5 FORMAT.

READIN = 1 IMPLIES ONLY ONE REPLICATION OF EACH JUDGMENT IN EACH DATA SET. THE ENTIRE CARD IS SKIPPED IF COLUMNS 1 AND 2 ARE EMPTY. THE NUMBER OF CARDS CANNOT EXCEED ONE PER CELL, AND NONE ARE NEEDED FOR EMPTY CELLS. THE CARDS WITHIN A DATA SET NEED NOT BE IN ORDER.

EACH DATA SET MUST BE FOLLOWED BY A CARD WITH "99" IN COLUMNS 1 AND 2 TO SIGNAL THE END OF THE DATA SET AND TO INITIATE THE ANALYSIS. THE NUMBER OF DISTINCT DATA SETS MUST EQUAL THE NUMBER INDICATED ON THE TASK CARD.

IF READIN = 2, THEN PROCEED AS FOLLOWS FOR READING IN THE DATA:

COLUMN	MEANING
1 - 2	DATA SET ID, FROM 01 TO 98.
4	NUMBER OF REPLICATIONS OF EACH JUDGMENT APPEARING ON EACH DATA CARD.

THIS PARAMETER CARD APPEARS BEFORE THE DATA CARDS. NOTE THAT FOR READIN = 2 IF IT ASSUMED THAT THE NUMBER OF REPLICATIONS OF EACH JUDGMENT IN THE DATA SET IS THE SAME. THE PARAMETER CARD IS FOLLOWED BY THE DATA CARDS. ALL REPLICATIONS OF EACH JUDGMENT ARE ON THE SAME CARD. OBSERVATIONS CAN BE PUNCHED FORMAT-FREE, EACH BEING SEPERATED BY AT LEAST ONE BLANK.

WHEN USING THIS SPECIAL READIN METHOD, HOWEVER, THE CARDS WITHIN EACH DATA SET MUST BE IN THE SAME SPECIAL ORDER. INDICES ON VARIABLES A, B, AND C NEED NOT BE PUNCHED. IN THIS INPUT MODE IT IS ASSUMED THAT THE DATA CARDS ARE IN THE FOLLOWING ORDER:

(1,1,1),(1,1,2),...,(1,1,Z),(1,2,1),(1,2,2),...
(1,2,Z),(1,3,1),...,(1,Y,Z),...,(X,Y,Z),
WHERE X, Y, AND Z ARE THE NUMBER OF LEVELS OF FACTORS A, B, AND C.

CARD 7. END OF DATA CARD.

A BLANK CARD FOLLOWS THE LAST DATA SET. THIS CARD SIGNALS THE END OF THE ANALYSIS.

```
**************************************************
*                                                *
*  C O N J O I N T . E X A M P L E               *
*                                                *
**************************************************
```

THE FOLLOWING CARDS ARE A SAMPLE DATA DECK FOR A "CONJOINT" ANALYSIS. IN THE EXAMPLE, "CONJOINT" IS USED TO CHECK ALL SIX TESTS (I.E., DOUBLE CANCELLATION) FOR THE DATA FROM ONE SUBJECT. THE STIMULUS DESIGN IS A THREE-FACTOR 4X3X4 DESIGN. THERE ARE THREE REPLICATIONS OF EACH OF THE 48 STIMULI. SPECIAL READ IN METHOD READIN=2 WAS USED. NOTE THAT INDICES FOR THE FACTOR LEVELS ARE NOT PUNCHED ON THE DATA CARDS. THE DATA ARE ASSUMED TO BE IN THE ORDER DESCRIBED IN THE SECTION ABOVE.

```
TASK 2 4 4 3 2 0 0
TITL 'ATTRACTIVENESS & RISK DATA, 1 SUB.  1X4X3X4 DESIGN.'
TEST 1 3 'A' 'B' 'C'
TEST 2 3 'A' 'B' 'C'
TEST 3 3 'A' 'B' 'C'
TEST 4 3 'A' 'B' 'C'
TEST 5 3 'A' 'B' 'C'
TEST 6 3 'A' 'B' 'C'
ALBL 'A' 'AMOUNT TO LOSE'
BLBL 'B' 'PROBABILITY OF LOSING'
CLBL 'C' 'PROBABILITY OF WINNING'
 1 3
    58    67    68
    77    78    74
    86    94    91
    56    57    64
    74    75    77
    85    89    91
    54    55    59
    64    71    73
    51    86    88
    79    89    89
    91    93    94
    48    69    67
    56    62    69
    68    73    72
    76    87    86
    81    49    49
    52    59    65
    76    78    77
    38    41    32
    52    52    53
    59    64    69
```

77	79	83
88	91	93
59	59	61
51	49	48
55	64	65
64	72	76
51	35	36
49	49	48
57	63	59
42	21	21
36	34	36
79	49	48
59	78	77
85	88	88
55	54	57
39	41	41
61	54	54
61	67	69
35	22	23
36	38	35
52	47	48
9	9	8
21	25	19
39	32	29
69	72	71
52	86	87
58	76	75

2 3

3	5	5
4	5	5
4	5	6
9	9	9
8	9	9
9	9	9
14	15	15
15	15	15
15	15	15
19	19	19
19	19	19
19	19	19
9	9	9
9	9	9
6	9	9
19	19	19
18	19	19
19	19	19
52	52	51
29	29	29
29	28	29

89	89	89
38	39	39
39	39	39
15	15	16
15	14	15
14	14	15
51	52	51
29	29	29
29	29	29
92	91	91
61	51	52
46	45	45
96	96	92
95	94	94
59	59	59
19	19	19
19	19	19
19	19	19
88	89	89
39	39	89
39	39	39
94	96	95
96	93	94
59	59	59
98	99	96
97	98	96
98	97	96

D I S T R I B

```
*****************************************************
*                                                   *
*  D   I   S   T   R   I   B                        *
*                                                   *
*  INSTRUCTIONS FOR USING DISTRIB                   *
*                                                   *
*****************************************************
*****************************************************
*                                                   *
*  THE PROGRAM HAS BEEN MODIFIED AND ADAPTED FOR    *
*  USE ON THE AMDAHL 470 OPERATING SYSTEM AT        *
*  THE OHIO STATE UNIVERSITY BY:                    *
*  DR. THOMAS E. NYGREN                             *
*  DEPT. OF PSYCHOLOGY                              *
*  OHIO STATE UNIVERSITY                            *
*                                                   *
*****************************************************
```

```
*************************************************************
*                                                           *
*  TO CALL THIS PROGRAM FOR THE SOURCE DECK FORM FROM       *
*  THE TAPE "CPSCAL", USE THE FOLLOWING JCL CARDS:          *
*                                                           *
*   1)  YOUR ID CARD                                        *
*   2)  //  TIME=2,REGION=400K                             *
*   3)  /*JOBPARM LINES=10000,DISKIO=500,TAPEIO=500        *
*   4)  /*SETUP UNIT=TAPE9,ID=(CPSCAL,XXXX,READ)          *
*   5)  //  EXEC FORTHCLG,TIME.GO=2,REGION.GO=400K        *
*   6)  //FORT.SYSIN DD UNIT=3400,VOL=SER=CPSCAL,         *
*   7)  //  DISP=(OLD,PASS),DSN=DISTRIB,                  *
*   8)  //  LABEL=(2,SL),                                 *
*   9)  //  DCB=(RECFM=FB,LRECL=80,BLKSIZE=1600)          *
*  10)  //GO.FT02F001  DD  UNIT=SYSDA,SPACE=(CYL,(1,1)),  *
*  11)  //  DCB=(RECFM=VBS,BLKSIZE=13030),                *
*  12)  //  DISP=(NEW,DELETE)                             *
*  13)  //GO.SYSIN DD *                                   *
*                                                           *
*  14)  INPUT DECK AS DESCRIBED BELOW (CONTROL AND          *
*       DATA CARDS GO HERE.)                                *
*                                                           *
*  15)  /*                                                 *
*  16)  //                                                *
*                                                           *
*************************************************************
```

INPUT DECK ARRANGEMENT

CARD(S) A. (JCL CARDS- SEE ABOVE CARDS 1 THROUGH 13.)

CARD B. TITLE CARD IDENTIFIER.
THIS IS A CARD USED TO PROVIDE A TITLE TO THE OUTPUT. "TITL" MUST BE PUNCHED IN COLUMNS 1-4.

CARD C. TITLE CARD. (FORMAT IS 18A4.) ANY INFORMATION MAY BE TYPED IN COLUMNS 1-72.

CARD(S) D. ADDITIONAL INFORMATION CARDS.
ANY OF THE FOLLOWING STATEMENT PHRASES MAY BE TYPED ON ONE OR MORE CARDS. THESE PHRASES CONTROL A NUMBER OF INPUT, OUTPUT, AND CONTROL PARAMETERS WITHIN THE DISTRIB PROGRAM. LEFT JUSTIFY EACH CARD BY BEGINNING IN COLUMN 1. THE PHRASES ARE:

PHRASE	MEANING
PRINT INPUT	INPUT DATA WILL BE PRINTED BACK OUT.
PRINT NOINPUT	INPUT DATA WILL NOT BE PRINTED OUT. THIS IS THE DEFAULT OPTION. IF THE "PRINT" STATEMENT IS OMITTED, THE DATA WILL NOT BE PRINTED OUT.
CUTOFF= X.X	X.X IS A DECIMAL NUMBER WHICH INDICATES THAT ALL DATA VALUES LESS THAN OR EQUAL TO IT WILL BE TREATED AS MISSING DATA. THIS VALUE IS SET TO -1.23 X 10**20 IF THIS CONTROL PHRASE IS OMITTED. SET THIS PARAMETER TO 0.0 IF ZEROS, BLANKS, AND/OR NEGATIVE NUMBERS ARE TO BE TREATED AS MISSING DATA.
NOCARDS	THE FINAL CONFIGURATION WILL NOT BE PUNCHED ON CARDS. THIS IS THE DEFAULT. THAT IS, IF THIS PHRASE IS OMITTED, NO CARDS WILL BE PUNCHED.
CARDS	THE FINAL CONFIGURATION WILL BE PUNCHED.
ITERATIONS= X	THE PROGRAM WILL USE A MAXIMUM OF "X" ITERATIONS TO ACHIEVE A FINAL CONFIGURATION WITH MINIMUM STRESS.

TIES PRIMARY	IF THERE ARE TIES IN THE DATA, THE PROGRAM CAN PERFORM TWO DIFFERENT MONOTONE REGRESSION PROCEDURES. WHEN THE PRIMARY APPROACH IS SPECIFIED, THE MONOTONE REGRESSION PROCEDURE IS NOT FORCED TO LEAVE TIES IN THE ORIGINAL DATA AS TIES IN THE FINAL SCALE VALUES. THAT IS, ALTHOUGH THE ORIGINAL DATA MAY HAVE EQUAL VALUES, THE FINAL SCALE VALUES NEED NOT BE EQUAL. THIS IS THE DEFAULT.
TIES SECONDARY	IF THERE ARE TWO OR MORE DATA VALUES THAT ARE TIED, THE MONOTONE REGRESSION PROCEDURE WILL KEEP THEIR CORRESPONDING FINAL SCALE VALUES TIED OR EQUAL. IF NEITHER OF THE "TIES" STATEMENTS ARE PRESENT, "TIES PRIMARY" IS ASSUMED. FOR SMALL DESIGNS IT IS STRONGLY RECOMMENDED THAT "TIES SECONDARY" BE USED IN ORDER TO REDUCE THE PROBABILITY OF GETTING A LOCAL MINIMUM SOLUTION.
* AAAAAAAAAAA	ANY CARD BEGINNING WITH A "*" IN COLUMN 1 SERVES AS A COMMENT CARD. THERE IS NO LIMIT AS TO THE NUMBER OF SUCH CARDS THAT CAN BE USED. THE COMMENT CAN GO UP THROUGH COLUMN 72. ON CARDS FOR WHICH THE "*" DOES NOT OCCUR IN COLUMN 1, ALL INFORMATION FROM THE POINT OF OCCURENCE OF THE "*" IS TREATED AS A COMMENT.

(CARD E.) INITIAL CONFIGURATION CARD. THIS CARD IS OPTIONAL. IT IS ONLY PRESENT IS AN INITIAL STIMULUS CONFIGURATION IS TO BE READ IN BY THE USER. IF SO, THE WORD "CONFIGURATION" SHOULD BE TYPED IN COLUMNS 1-13.

(CARD F.) PARAMETER CARD FOR INITIAL CONFIGURATION. THIS CARD IS OPTIONAL AND IS PRESENT ONLY IF CARD E IS PRESENT. THIS CARD DESCRIBES THE LEVELS OF THE FACTORS IN THE INITIAL CONFIGURATION. FORMAT IS 4I4.

COL.	PARAMETER	MEANING
1- 4	KK	NUMBER OF FACTORS. THIS VALUE MUST BE 3.
5- 8	II(1)	NUMBER OF LEVELS OF FACTOR 1.
9-12	II(2)	NUMBER OF LEVELS OF FACTOR 2.
13-16	II(3)	NUMBER OF LEVELS OF FACTOR 3.

(CARD G.) FORMAT FOR INITIAL CONFIGURATION. THIS CARD IS ALSO OPTIONAL AND IS PRESENT ONLY IF CARD E IS PRESENT. THIS CARD INDICATES THE FORMAT FOR READING IN THE INITIAL VALUES OF THE LEVELS OF EACH OF THE FACTORS. EACH SET OF VALUES FOR THE FACTORS IS ASSUMED TO START ON A NEW CARD. THE FORMAT STATEMENT CAN BE UP TO 80 CHARACTERS IN LENGTH AND SHOULD BEGIN AND END WITH PARENTHESES. FOR EXAMPLE, (5F8.4).

(CARD(S) H.) INITIAL CONFIGURATION.
THESE CARDS ARE PRESENT ONLY IF CARD E IS PRESENT. CARDS H ARE THE INITIAL CONFIGURATION PUNCHED AS SPECIFIED ON CARD G.

CARD I. DATA CONTROL CARD.
THE WORD "DATA" IS PUNCHED IN COLUMNS 1-4 TO INDICATE THAT THE PARAMETER, FORMAT AND ACTUAL DATA CARDS WILL FOLLOW.

CARD J. PARAMETER CARD. FORMAT IS 9I4.

COL.	PARAMETER	MEANING
1- 4	KK	NUMBER OF FACTORS. THIS VALUE MUST BE 3.
5- 8	II(1)	NUMBER OF LEVELS OF FACTOR 1.
9-12	II(2)	NUMBER OF LEVELS OF FACTOR 2.
13-16	II(3)	NUMBER OF LEVELS OF FACTOR 3.
17-20	LREPLI	NUMBER OF REPLICATIONS OF EACH OBSERVATION IN THE DATA SET.
21-24	NREVL	=0, IF THE INPUT DATA WILL BE GIVEN VALUES OF THE OPPOSITE SIGN. THAT IS, THE DATA WILL BE REVERSED. =1, IF THE INPUT DATA WILL BE LEFT AS IS.
25-28	JUNIT	=0, INPUT DATA VALUES ARE ON PUNCHED CARDS. =N, INPUT DATA VALUES ARE ON LOGICAL UNIT NUMBER 'N'.

CARD K. FORMAT FOR READING IN THE DATA.
THIS CARD INDICATES THE FORMAT FOR READING IN THE DATA. THE DATA ARE ASSUMED TO BE IN THE NATURAL ORDER. FOR EXAMPLE, IN A THREE FACTOR DESIGN THIS WOULD IMPLY THAT ALL REPLICATIONS OF CELL (1,1,1) WOULD BE READ FIRST, FOLLOWED BY (1,1,2), (1,1,3) AND SO ON. THUS, FACTOR 3 VARIES FASTEST, FOLLOWED BY FACTORS 2 AND 1. THE ACTUAL FORMAT IS FLEXIBLE. THERE MAY BE ONE OBSERVATION PER CARD, ALL REPLICATIONS OF A CELL ON A CARD, OR ALL OBSERVATIONS STRUNG OUT ON THE CARD(S).

CARD(S) L. DATA CARD(S) IN THE FORMAT SPECIFIED IN CARD K.

CARD M. COMPUTE CARD.
THE WORD "COMPUTE" IS PUNCHED IN COLUMNS 1 - 7 TO INDICATE THAT THE NECESSARY INFORMATION HAS BEEN PROVIDED TO BEGIN THE COMPUTATION. EACH TIME A COMPUTE CARD IS ENCOUNTERED A NEW ANALYSIS IS BEGUN.

CARD N. STOP CARD.
THE WORD "STOP" IS PUNCHED IN COLUMNS 1 - 4 TO INDICATE THE END OF ALL ANALYSES. MULTIPLE ANALYSES CAN BE DONE IN EACH COMPUTER RUN. ONLY THE LAST ANALYSIS SHOULD BE CONCLUDED WITH A STOP CARD.

```
*****************************************************
*                                                   *
*  D I S T R I B . E X A M P L E                    *
*                                                   *
*****************************************************

THE FOLLOWING DATA CARDS ILLUSTRATE FOUR EXAMPLES
OF USING THE DISTRIB PROGRAM.  THE DATA IN EXAMPLES 2 AND 3 ARE
EXACTLY THE SAME AS THOSE USED IN MONANOVA.

TITLE
DISTRIBUTIVE MODEL
PRINT INPUT
TIES SECONDARY
*EXAMPLE NO. 1.
*
*EXAMPLE OF DISTRIBUTIVE MODEL
*GAMBLING DATA
*4X4X3 DESIGN.   ATTRACTIVENESS DATA FOR SUBJECT 1.
STRMIN=0.0001
ITERATIONS=30
DATA
   3   4   4   3   3   0   0
(3F6.0)
    58    67    68
    77    78    74
    86    94    91
    56    57    64
    74    75    77
    85    89    91
    54    55    59
    64    71    73
    51    86    88
    79    89    89
    91    93    94
    48    69    67
    56    62    69
    68    73    72
    76    87    86
    81    49    49
    52    59    65
    76    78    77
    38    41    32
    52    52    53
    59    64    69
    77    79    83
    88    91    93
    59    59    61
    51    49    48
```

```
     55     64     65
     64     72     76
     51     35     36
     49     49     48
     57     63     59
     42     21     21
     36     34     36
     79     49     48
     59     78     77
     85     88     88
     55     54     57
     39     41     41
     61     54     54
     61     67     69
     35     22     23
     36     38     35
     52     47     48
      9      9      8
     21     25     19
     39     32     29
     69     72     71
     52     86     87
     58     76     75
COMPUTE
TITLE
DISTRIBUTIVE MODEL
PRINT INPUT
TIES SECONDARY
*EXAMPLE NO. 2.
*
*EXAMPLE OF DISTRIBUTIVE MODEL
*GAMBLING DATA
*4X4X3 DESIGN.   RISKINESS DATA FOR SUBJECT 1.
STRMIN=0.0001
ITERATIONS=30
DATA
   3   4   4   3   3   0   0
(3F6.0)
      3      5      5
      4      5      5
      4      5      6
      9      9      9
      8      9      9
      9      9      9
     14     15     15
     15     15     15
     15     15     15
     19     19     19
```

```
     19     19     19
     19     19     19
      9      9      9
      9      9      9
      6      9      9
     19     19     19
     18     19     19
     19     19     19
     52     52     51
     29     29     29
     29     28     29
     89     89     89
     38     39     39
     39     39     39
     15     15     16
     15     14     15
     14     14     15
     51     52     51
     29     29     29
     29     29     29
     92     91     91
     61     51     52
     46     45     45
     96     96     92
     95     94     94
     59     59     59
     19     19     19
     19     19     19
     19     19     19
     88     89     89
     39     39     89
     39     39     39
     94     96     95
     96     93     94
     59     59     59
     98     99     96
     97     98     96
     98     97     96
COMPUTE
TITLE
DISTRIBUTIVE MODEL
PRINT INPUT
TIES SECONDARY
*EXAMPLE NO. 3.
*
* PERFECT DISTRIBUTIVE DATA.
* 4X3X2 DESIGN.
STRMIN=0.0001
ITERATIONS=30
```

```
DATA
   3   4   3   2   1   0   0
(10F4.0)
   3   4   5   6   7   8   9  12  15  18
  21  24  15  20  25  30  35  40  21  28
  35  42  49  56
COMPUTE
TITLE
DISTRIBUTIVE MODEL, MISSING DATA
PRINT INPUT
TIES SECONDARY
*EXAMPLE NO. 4.
*
* PERFECT DISTRIBUTIVE DATA, MISSING OBSERVATIONS.
* 4X3X2 DESIGN.
STRMIN=0.0001
ITERATIONS=30
DATA
   3   4   3   2   1   0   0
(10F4.0)
   3   4   5   6  -7   8   9  12  15  18
  21  24  15  20  25  30  35 -40  21  28
  35  42  49  56
COMPUTE
STOP
```

D U A L D I S T

```
**************************************************
*                                                *
*  D   U   A   L   D   I   S   T                 *
*                                                *
*  INSTRUCTIONS FOR USING DUALDIST               *
*                                                *
**************************************************
**************************************************
*                                                *
*  THE PROGRAM HAS BEEN MODIFIED AND ADAPTED FOR *
*  USE ON THE AMDAHL 470 OPERATING SYSTEM AT     *
*  THE OHIO STATE UNIVERSITY BY:                 *
*  DR. THOMAS E. NYGREN                          *
*  DEPT. OF PSYCHOLOGY                           *
*  OHIO STATE UNIVERSITY                         *
*                                                *
**************************************************

****************************************************************
*                                                              *
*  TO CALL THIS PROGRAM FOR THE SOURCE DECK FORM FROM          *
*  THE TAPE "CPSCAL", USE THE FOLLOWING JCL CARDS:             *
*                                                              *
*   1)  YOUR ID CARD                                           *
*   2)  //  TIME=2,REGION=400K                                 *
*   3)  /*JOBPARM LINES=10000,DISKIO=500,TAPEIO=500           *
*   4)  /*SETUP UNIT=TAPE9,ID=(CPSCAL,XXXX,READ)             *
*   5)  //  EXEC FORTHCLG,TIME.GO=2,REGION.GO=400K
*   6)  //FORT.SYSIN DD UNIT=3400,VOL=SER=CPSCAL,            *
*   7)  //  DISP=(OLD,PASS),DSN=DUALDIST,                    *
*   8)  //  LABEL=(3,SL),                                    *
*   9)  //  DCB=(RECFM=FB,LRECL=80,BLKSIZE=1600)             *
*  10)  //GO.FT02F001  DD  UNIT=SYSDA,SPACE=(CYL,(1,1)),     *
*  11)  //  DCB=(RECFM=VBS,BLKSIZE=13030),                   *
*  12)  //  DISP=(NEW,DELETE)                                *
*  13)  //GO.SYSIN DD *                                      *
*                                                           *
*  14)  INPUT DECK AS DESCRIBED BELOW (CONTROL AND          *
*       DATA CARDS GO HERE.)                                *
*                                                           *
*  15)  /*                                                  *
*  16)  //                                                  *
*                                                           *
****************************************************************
```

****

*

* I N P U T D E C K A R R A N G E M E N T *

*

****

CARD(S) A. (JCL CARDS- SEE ABOVE CARDS 1 THROUGH 13.)

CARD B. TITLE CARD IDENTIFIER.
THIS IS A CARD USED TO PROVIDE A TITLE TO THE OUTPUT.
"TITL" MUST BE PUNCHED IN COLUMNS 1-4.

CARD C. TITLE CARD. (FORMAT IS 18A4.) ANY INFORMATION MAY BE TYPED IN COLUMNS 1-72.

CARD(S) D. ADDITIONAL INFORMATION CARDS.
ANY OF THE FOLLOWING STATEMENT PHRASES MAY BE TYPED ON ONE OR MORE CARDS. THESE PHRASES CONTROL A NUMBER OF INPUT, OUTPUT, AND CONTROL PARAMETERS WITHIN THE DISTRIB PROGRAM. LEFT JUSTIFY EACH CARD BY BEGINNING IN COLUMN 1. THE PHRASES ARE:

PHRASE	MEANING
PRINT INPUT	INPUT DATA WILL BE PRINTED BACK OUT.
PRINT NOINPUT	INPUT DATA WILL NOT BE PRINTED OUT. THIS IS THE DEFAULT OPTION. IF THE "PRINT" STATEMENT IS OMITTED, THE DATA WILL NOT BE PRINTED OUT.
CUTOFF= X.X	X.X IS A DECIMAL NUMBER WHICH INDICATES THAT ALL DATA VALUES LESS THAN OR EQUAL TO IT WILL BE TREATED AS MISSING DATA. THIS VALUE IS SET TO -1.23 X 10**20 IF THIS CONTROL PHRASE IS OMITTED. SET THIS PARAMETER TO 0.0 IF ZEROS, BLANKS, AND/OR NEGATIVE NUMBERS ARE TO BE TREATED AS MISSING DATA.
NOCARDS	THE FINAL CONFIGURATION WILL NOT BE PUNCHED ON CARDS. THIS IS THE DEFAULT. THAT IS, IF THIS PHRASE IS OMITTED, NO CARDS WILL BE PUNCHED.
CARDS	THE FINAL CONFIGURATION WILL BE PUNCHED.
ITERATIONS= X	THE PROGRAM WILL USE A MAXIMUM OF "X" ITERATIONS TO ACHIEVE A FINAL CONFIGURATION WITH MINIMUM STRESS.

TIES PRIMARY	IF THERE ARE TIES IN THE DATA, THE PROGRAM CAN PERFORM TWO DIFFERENT MONOTONE REGRESSION PROCEDURES. WHEN THE PRIMARY APPROACH IS SPECIFIED, THE MONOTONE REGRESSION PROCEDURE IS NOT FORCED TO LEAVE TIES IN THE ORIGINAL DATA AS TIES IN THE FINAL SCALE VALUES. THAT IS, ALTHOUGH THE ORIGINAL DATA MAY HAVE EQUAL VALUES, THE FINAL SCALE VALUES NEED NOT BE EQUAL. THIS IS THE DEFAULT.
TIES SECONDARY	IF THERE ARE TWO OR MORE DATA VALUES THAT ARE TIED, THE MONOTONE REGRESSION PROCEDURE WILL KEEP THEIR CORRESPONDING FINAL SCALE VALUES TIED OR EQUAL. IF NEITHER OF THE "TIES" STATEMENTS ARE PRESENT, "TIES PRIMARY" IS ASSUMED. FOR SMALL DESIGNS IT IS STRONGLY RECOMMENDED THAT "TIES SECONDARY" BE USED IN ORDER TO REDUCE THE PROBABILITY OF GETTING A LOCAL MINIMUM SOLUTION.
* AAAAAAAAAAA	ANY CARD BEGINNING WITH A "*" IN COLUMN 1 SERVES AS A COMMENT CARD. THERE IS NO LIMIT AS TO THE NUMBER OF SUCH CARDS THAT CAN BE USED. THE COMMENT CAN GO UP THROUGH COLUMN 72. ON CARDS FOR WHICH THE "*" DOES NOT OCCUR IN COLUMN 1, ALL INFORMATION FROM THE POINT OF OCCURENCE OF THE "*" IS TREATED AS A COMMENT.

(CARD E.) INITIAL CONFIGURATION CARD. THIS CARD IS OPTIONAL. IT IS ONLY PRESENT IS AN INITIAL STIMULUS CONFIGURATION IS TO BE READ IN BY THE USER. IF SO, THE WORD "CONFIGURATION" SHOULD BE TYPED IN COLUMNS 1-13.

(CARD F.) PARAMETER CARD FOR INITIAL CONFIGURATION. THIS CARD IS OPTIONAL AND IS PRESENT ONLY IF CARD E IS PRESENT. THIS CARD DESCRIBES THE LEVELS OF THE FACTORS IN THE INITIAL CONFIGURATION. FORMAT IS 4I4.

COL.	PARAMETER	MEANING
1- 4	KK	NUMBER OF FACTORS. THIS VALUE MUST BE 3.
5- 8	II(1)	NUMBER OF LEVELS OF FACTOR 1.
9-12	II(2)	NUMBER OF LEVELS OF FACTOR 2.
13-16	II(3)	NUMBER OF LEVELS OF FACTOR 3.

(CARD G.) FORMAT FOR INITIAL CONFIGURATION. THIS CARD IS ALSO OPTIONAL AND IS PRESENT ONLY IF CARD E IS PRESENT. THIS CARD INDICATES THE FORMAT FOR READING IN THE INITIAL VALUES OF THE LEVELS OF EACH OF THE FACTORS. EACH SET OF VALUES FOR THE FACTORS IS ASSUMED TO START ON A NEW CARD. THE FORMAT STATEMENT CAN BE UP TO 80 CHARACTERS IN LENGTH AND SHOULD BEGIN AND END WITH PARENTHESES. FOR EXAMPLE, (5F8.4).

(CARD(S) H.) INITIAL CONFIGURATION.
THESE CARDS ARE PRESENT ONLY IF CARD E IS PRESENT. CARDS H ARE THE INITIAL CONFIGURATION PUNCHED AS SPECIFIED ON CARD G.

CARD I. DATA CONTROL CARD.
THE WORD "DATA" IS PUNCHED IN COLUMNS 1-4 TO INDICATE THAT THE PARAMETER, FORMAT AND ACTUAL DATA CARDS WILL FOLLOW.

CARD J. PARAMETER CARD. FORMAT IS 9I4.

COL.	PARAMETER	MEANING
1- 4	KK	NUMBER OF FACTORS. THIS VALUE MUST BE 3.
5- 8	II(1)	NUMBER OF LEVELS OF FACTOR 1.
9-12	II(2)	NUMBER OF LEVELS OF FACTOR 2.
13-16	II(3)	NUMBER OF LEVELS OF FACTOR 3.
17-20	LREPLI	NUMBER OF REPLICATIONS OF EACH OBSERVATION IN THE DATA SET.
21-24	NREVL	=0, IF THE INPUT DATA WILL BE GIVEN VALUES OF THE OPPOSITE SIGN. THAT IS, THE DATA WILL BE REVERSED. =1, IF THE INPUT DATA WILL BE LEFT AS IS.
25-28	JUNIT	=0, INPUT DATA VALUES ARE ON PUNCHED CARDS. =N, INPUT DATA VALUES ARE ON LOGICAL UNIT NUMBER 'N'.

CARD K. FORMAT FOR READING IN THE DATA.
THIS CARD INDICATES THE FORMAT FOR READING IN THE DATA. THE DATA ARE ASSUMED TO BE IN THE NATURAL ORDER. FOR EXAMPLE, IN A THREE FACTOR DESIGN THIS WOULD IMPLY THAT ALL REPLICATIONS OF CELL (1,1,1) WOULD BE READ FIRST, FOLLOWED BY (1,1,2), (1,1,3) AND SO ON. THUS, FACTOR 3 VARIES FASTEST, FOLLOWED BY FACTORS 2 AND 1. THE ACTUAL FORMAT IS FLEXIBLE. THERE MAY BE ONE OBSERVATION PER CARD, ALL REPLICATIONS OF A CELL ON A CARD, OR ALL OBSERVATIONS STRUNG OUT ON THE CARD(S).

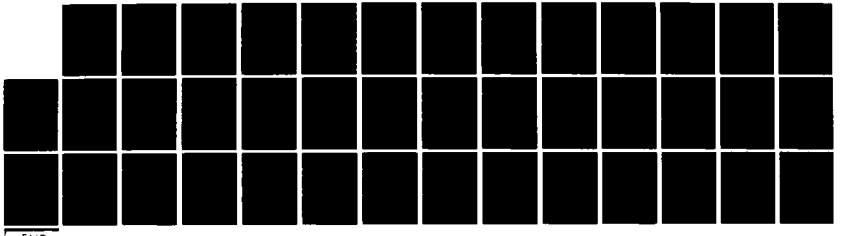
CARD(S) L. DATA CARD(S) IN THE FORMAT SPECIFIED IN CARD K.

CARD M. COMPUTE CARD.
THE WORD "COMPUTE" IS PUNCHED IN COLUMNS 1 - 7 TO INDICATE THAT THE NECESSARY INFORMATION HAS BEEN PROVIDED TO BEGIN THE COMPUTATION. EACH TIME A COMPUTE CARD IS ENCOUNTERED A NEW ANALYSIS IS BEGUN.

CARD N. STOP CARD.
THE WORD "STOP" IS PUNCHED IN COLUMNS 1 - 4 TO INDICATE THE END OF ALL ANALYSES. MULTIPLE ANALYSES CAN BE DONE IN EACH COMPUTER RUN. ONLY THE LAST ANALYSIS SHOULD BE CONCLUDED WITH A STOP CARD.

AD-A122 579 CONJOINT MEASUREMENT AND CONJOINT SCALING: A USERS GUIDE(U) AIR FORCE AEROSPACE MEDICAL RESEARCH LAB WRIGHT-PATTERSON AFB OH T E NYGREN APR 82 2/2

UNCLASSIFIED AFAMRL-TR-82-22 F/G 12/1 NL



END

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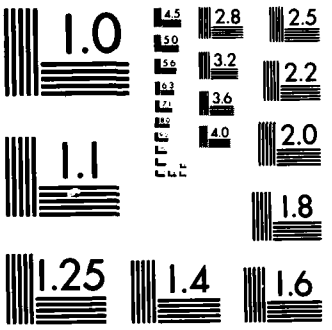
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

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**************************************************
*                                                *
*  D U A L D I S T . E X A M P L E               *
*                                                *
**************************************************

THE FOLLOWING DATA CARDS INDICATE FOUR EXAMPLES
OF USING THE DUALDIST PROGRAM.  THE DATA IN EXAMPLES
1 AND 2 ARE EXACTLY THE SAME AS THOSE USED IN MONANOVA.

TITLE
DUAL DISTRIBUTIVE MODEL
PRINT INPUT
TIES SECONDARY
*EXAMPLE NO. 1.
*
*EXAMPLE OF DUAL DISTRIBUTIVE MODEL
*GAMBLING DATA
*4X4X3 DESIGN.   ATTRACTIVENESS DATA FOR SUBJECT 1.
STRMIN=0.0001
ITERATIONS=30
DATA
   3   4   4   3   3   0   0
(3F6.0)
    58    67    68
    77    78    74
    86    94    91
    56    57    64
    74    75    77
    85    89    91
    54    55    59
    64    71    73
    51    86    88
    79    89    89
    91    93    94
    48    69    67
    56    62    69
    68    73    72
    76    87    86
    81    49    49
    52    59    65
    76    78    77
    38    41    32
    52    52    53
    59    64    69
    77    79    83
    88    91    93
    59    59    61
    51    49    48
```

```
     55     64     65
     64     72     76
     51     35     36
     49     49     48
     57     63     59
     42     21     21
     36     34     36
     79     49     48
     59     78     77
     85     88     88
     55     54     57
     39     41     41
     61     54     54
     61     67     69
     35     22     23
     36     38     35
     52     47     48
      9      9      8
     21     25     19
     39     32     29
     69     72     71
     52     86     87
     58     76     75
COMPUTE
TITLE
DUAL DISTRIBUTIVE MODEL
PRINT INPUT
TIES SECONDARY
*EXAMPLE NO. 2.
*
*GAMBLING DATA
*4X4X3 DESIGN.    RISKINESS DATA FOR SUBJECT 1.
STRMIN=0.0001
ITERATIONS=30
DATA
   3   4   4   3   3   0   0
(3F6.0)
      3      5      5
      4      5      5
      4      5      6
      9      9      9
      8      9      9
      9      9      9
     14     15     15
     15     15     15
     15     15     15
     19     19     19
     19     19     19
     19     19     19
```

```
      9      9      9
      9      9      9
      6      9      9
     19     19     19
     18     19     19
     19     19     19
     52     52     51
     29     29     29
     29     28     29
     89     89     89
     38     39     39
     39     39     39
     15     15     16
     15     14     15
     14     14     15
     51     52     51
     29     29     29
     29     29     29
     92     91     91
     61     51     52
     46     45     45
     96     96     92
     95     94     94
     59     59     59
     19     19     19
     19     19     19
     19     19     19
     88     89     89
     39     39     89
     39     39     39
     94     96     95
     96     93     94
     59     59     59
     98     99     96
     97     98     96
     98     97     96
COMPUTE
TITLE
DUAL DISTRIBUTIVE MODEL
PRINT INPUT
TIES SECONDARY
*EXAMPLE NO. 3.
*
* PERFECT DUAL DISTRIBUTIVE DATA.
* 4X3X2 DESIGN.
STRMIN=0.0001
ITERATIONS=30
DATA
```

```
   3   4   3   2   1   0   0
(10F4.0)
   3   5   5   9   7  13   5   7   7  11
   9  15   7   9   9  13  11  17   9  11
  11  15  13  19
COMPUTE
TITLE
DUAL-DISTRIBUTIVE MODEL, MISSING DATA
PRINT INPUT
TIES SECONDARY
*EXAMPLE NO. 4.
*
* PERFECT DUAL-DISTRIBUTIVE DATA, MISSING OBSERVATIONS.
* 4X3X2 DESIGN.
STRMIN=0.0001
ITERATIONS=30
DATA
   3   4   3   2   1   0   0
(10F4.0)
   3   5   5   9  -7  13   5   7   7  11
   9  15   7   9   9  13 -11  17   9  11
  11  15  13  19
COMPUTE
STOP
```

M O N A N O V A

```
**************************************************
*                                                *
*  M   O   N   A   N   O   V   A                 *
*                                                *
*  INSTRUCTIONS FOR USING MONANOVA               *
*                                                *
**************************************************
**************************************************
*                                                *
*  THE PROGRAM HAS BEEN MODIFIED AND ADAPTED FOR *
*  USE ON THE AMDAHL 470 OPERATING SYSTEM AT     *
*  THE OHIO STATE UNIVERSITY BY:                 *
*  DR. THOMAS E. NYGREN                          *
*  DEPT. OF PSYCHOLOGY                           *
*  OHIO STATE UNIVERSITY                         *
*                                                *
**************************************************

**************************************************************
*                                                            *
*  TO CALL THIS PROGRAM FOR THE SOURCE DECK FORM FROM        *
*  THE TAPE "CPSCAL", USE THE FOLLOWING JCL CARDS:           *
*                                                            *
*  1)  YOUR ID CARD                                          *
*  2)  //  TIME=2,REGION=400K                                *
*  3)  /*JOBPARM LINES=10000,DISKIO=500,TAPEIO=500          *
*  4)  /*SETUP UNIT=TAPE9,ID=(CPSCAL,XXXX,READ)            *
*  5)  //  EXEC FORTHCLG,TIME.GO=2,REGION.GO=400K          *
*  6)  //FORT.SYSIN DD UNIT=3400,VOL=SER=CPSCAL,           *
*  7)  //  DISP=(OLD,PASS),DSN=MONANOVA,                   *
*  8)  //  LABEL=(4,SL),                                   *
*  9)  //  DCB=(RECFM=FB,LRECL=80,BLKSIZE=1600)            *
* 10)  //GO.FT02F001  DD  UNIT=SYSDA,SPACE=(CYL,(1,1)),    *
* 11)  //  DCB=(RECFM=VBS,BLKSIZE=13030),                  *
* 12)  //  DISP=(NEW,DELETE)                               *
* 13)  //GO.SYSIN DD *                                     *
*                                                          *
* 14)  INPUT DECK AS DESCRIBED BELOW (CONTROL AND          *
*      DATA CARDS GO HERE.)                                *
*                                                          *
* 15)  /*                                                  *
* 16)  //                                                 *
*                                                          *
************************************************************
```

```
**************************************************
*                                                *
*  I N P U T   D E C K   A R R A N G E M E N T   *
*                                                *
**************************************************
```

CARD(S) A. (JCL CARDS- SEE ABOVE CARDS 1 THROUGH 13.)

CARD B. TITLE CARD IDENTIFIER.
THIS IS A CARD USED TO PROVIDE A TITLE TO THE OUTPUT.
"TITL" MUST BE PUNCHED IN COLUMNS 1-4.

CARD C. TITLE CARD. (FORMAT IS 18A4.) ANY INFORMATION MAY BE TYPED IN COLUMNS 1-72.

CARD(S) D. ADDITIONAL INFORMATION CARDS.
ANY OF THE FOLLOWING STATEMENT PHRASES MAY BE TYPED ON ONE OR MORE CARDS. THESE PHRASES CONTROL A NUMBER OF INPUT, OUTPUT, AND CONTROL PARAMETERS WITHIN THE MONANOVA PROGRAM. LEFT JUSTIFY EACH CARD BY BEGINNING IN COLUMN 1. THE PHRASES ARE:

PHRASE	MEANING
PRINT INPUT	INPUT DATA WILL BE PRINTED BACK OUT.
PRINT NOINPUT	INPUT DATA WILL NOT BE PRINTED OUT. THIS IS THE DEFAULT OPTION. IF THE "PRINT" STATEMENT IS OMITTED, THE DATA WILL NOT BE PRINTED OUT.
CUTOFF= X.X	X.X IS A DECIMAL NUMBER WHICH INDICATES THAT ALL DATA VALUES LESS THAN OR EQUAL TO IT WILL BE TREATED AS MISSING DATA. THIS VALUE IS SET TO -1.23 X 10**20 IF THIS CONTROL PHRASE IS OMITTED. SET THIS PARAMETER TO 0.0 IF ZEROS, BLANKS, AND/OR NEGATIVE NUMBERS ARE TO BE TREATED AS MISSING DATA.
NOCARDS	THE FINAL CONFIGURATION WILL NOT BE PUNCHED ON CARDS. THIS IS THE DEFAULT. THAT IS, IF THIS PHRASE IS OMITTED, NO CARDS WILL BE PUNCHED.
CARDS	THE FINAL CONFIGURATION WILL BE PUNCHED.
ITERATIONS= X	THE PROGRAM WILL USE A MAXIMUM OF "X" ITERATIONS TO ACHIEVE A FINAL CONFIGURATION WITH MINIMUM STRESS.

TIES PRIMARY	IF THERE ARE TIES IN THE DATA, THE PROGRAM CAN PERFORM TWO DIFFERENT MONOTONE REGRESSION PROCEDURES. WHEN THE PRIMARY APPROACH IS SPECIFIED, THE MONOTONE REGRESSION PROCEDURE IS NOT FORCED TO LEAVE TIES IN THE ORIGINAL DATA AS TIES IN THE FINAL SCALE VALUES. THAT IS, ALTHOUGH THE ORIGINAL DATA MAY HAVE EQUAL VALUES, THE FINAL SCALE VALUES NEED NOT BE EQUAL. THIS IS THE DEFAULT.
TIES SECONDARY	IF THERE ARE TWO OR MORE DATA VALUES THAT ARE TIED, THE MONOTONE REGRESSION PROCEDURE WILL KEEP THEIR CORRESPONDING FINAL SCALE VALUES TIED OR EQUAL. IF NEITHER OF THE "TIES" STATEMENTS ARE PRESENT, "TIES PRIMARY" IS ASSUMED. FOR SMALL DESIGNS IT IS STRONGLY RECOMMENDED THAT "TIES SECONDARY" BE USED IN ORDER TO REDUCE THE PROBABILITY OF GETTING A LOCAL MINIMUM SOLUTION.
* AAAAAAAAAAA	ANY CARD BEGINNING WITH A "*" IN COLUMN 1 SERVES AS A COMMENT CARD. THERE IS NO LIMIT AS TO THE NUMBER OF SUCH CARDS THAT CAN BE USED. THE COMMENT CAN GO UP THROUGH COLUMN 72. ON CARDS FOR WHICH THE "*" DOES NOT OCCUR IN COLUMN 1, ALL INFORMATION FROM THE POINT OF OCCURENCE OF THE "*" IS TREATED AS A COMMENT.

(CARD E.) INITIAL CONFIGURATION CARD. THIS CARD IS OPTIONAL. IT IS ONLY PRESENT IS AN INITIAL STIMULUS CONFIGURATION IS TO BE READ IN BY THE USER. IF SO, THE WORD "CONFIGURATION" SHOULD BE TYPED IN COLUMNS 1-13.

(CARD F.) PARAMETER CARD FOR INITIAL CONFIGURATION. THIS CARD IS OPTIONAL AND IS PRESENT ONLY IF CARD E IS PRESENT. THIS CARD DESCRIBES THE LEVELS OF THE FACTORS IN THE INITIAL CONFIGURATION. FORMAT IS 6I4.

COL.	PARAMETER	MEANING
1- 4	KK	NUMBER OF FACTORS. MAX=5.
5- 8	II(1)	NUMBER OF LEVELS OF FACTOR 1.
9-12	II(2)	NUMBER OF LEVELS OF FACTOR 2.
13-16	II(3)	NUMBER OF LEVELS OF FACTOR 3.
17-20	II(4)	NUMBER OF LEVELS OF FACTOR 4.
21-24	II(5)	NUMBER OF LEVELS OF FACTOR 5. WHEN THE NUMBER OF FACTORS IS < 5, THE NUMBER OF LEVELS OF THE HIGHER FACTORS SHOULD BE TYPED AS "1" IN THE APPROPRIATE COLUMN(S).

(CARD G.) FORMAT FOR INITIAL CONFIGURATION. THIS CARD IS ALSO OPTIONAL AND IS PRESENT ONLY IF CARD E IS PRESENT. THIS CARD INDICATES THE FORMAT FOR READING IN THE INITIAL VALUES OF THE LEVELS OF EACH OF THE FACTORS. EACH SET OF VALUES FOR THE FACTORS IS ASSUMED TO START ON A NEW CARD. THE FORMAT STATEMENT CAN BE UP TO 80 CHARACTERS IN LENGTH AND SHOULD BEGIN AND END WITH PARENTHESES. FOR EXAMPLE, (5F8.4).

(CARD(S) H.) INITIAL CONFIGURATION. THESE CARDS ARE PRESENT ONLY IF CARD E IS PRESENT. CARDS H ARE THE INITIAL CONFIGURATION PUNCHED AS SPECIFIED ON CARD G.

CARD I. DATA CONTROL CARD. THE WORD "DATA" IS PUNCHED IN COLUMNS 1-4 TO INDICATE THAT THE PARAMETER, FORMAT AND ACTUAL DATA CARDS WILL FOLLOW.

CARD J. PARAMETER CARD. (FORMAT IS 9I4.)

COL.	PARAMETER	MEANING
1- 4	KK	NUMBER OF FACTORS. MAX=5.
5- 8	II(1)	NUMBER OF LEVELS OF FACTOR 1.
9-12	II(2)	NUMBER OF LEVELS OF FACTOR 2.
13-16	II(3)	NUMBER OF LEVELS OF FACTOR 3.
17-20	II(4)	NUMBER OF LEVELS OF FACTOR 4.

21-24	II(5)	NUMBER OF LEVELS OF FACTOR 5. WHEN THE NUMBER OF FACTORS IS < 5, THE NUMBER OF LEVELS OF THE HIGHER FACTORS SHOULD BE TYPED AS "1" IN THE APPROPRIATE COLUMN(S).
25-28	LREPLI	NUMBER OF REPLICATIONS OF EACH OBSERVATION IN THE DATA SET.
29-32	NREVL	=0, IF THE INPUT DATA WILL BE GIVEN VALUES OF THE OPPOSITE SIGN. THAT IS, THE DATA WILL BE REVERSED. =1, IF THE INPUT DATA WILL BE LEFT AS IS.
33-36	JUNIT	=0, INPUT DATA VALUES ARE ON PUNCHED CARDS. =N, INPUT DATA VALUES ARE ON LOGICAL UNIT NUMBER 'N'.

CARD K. FORMAT FOR READING IN THE DATA.
THIS CARD INDICATES THE FORMAT FOR READING IN THE DATA. THE DATA ARE ASSUMED TO BE IN THE NATURAL ORDER. FOR EXAMPLE, IN A THREE FACTOR DESIGN THIS WOULD IMPLY THAT ALL REPLICATIONS OF CELL (1,1,1) WOULD BE READ FIRST, FOLLOWED BY (1,1,2), (1,1,3) AND SO ON. THUS, FACTOR 3 VARIES FASTEST, FOLLOWED BY FACTORS 2 AND 1. THE ACTUAL FORMAT IS FLEXIBLE. THERE MAY BE ONE OBSERVATION PER CARD, ALL REPLICATIONS OF A CELL ON A CARD, OR ALL OBSERVATIONS STRUNG OUT ON THE CARD(S).

CARD(S) L. DATA CARD(S) IN THE FORMAT SPECIFIED IN

CARD M. COMPUTE CARD.

THE WORD "COMPUTE" IS PUNCHED IN COLUMNS 1 - 7 TO INDICATE THAT THE NECESSARY INFORMATION HAS BEEN PROVIDED TO BEGIN THE COMPUTATION. EACH TIME A COMPUTE CARD IS ENCOUNTERED A NEW ANALYSIS IS BEGUN.

CARD N. STOP CARD.

THE WORD "STOP" IS PUNCHED IN COLUMNS 1 - 4 TO INDICATE THE END OF ALL ANALYSES. MULTIPLE ANALYSES CAN BE DONE IN EACH COMPUTER RUN. ONLY THE LAST ANALYSIS SHOULD BE CONCLUDED WITH A STOP CARD.

```
**************************************************
*                                                *
*  M O N A N O V A . E X A M P L E               *
*                                                *
**************************************************

THE FOLLOWING DATA CARDS ILLUSTRATE SEVERAL EXAMPLES
OF USING MONANOVA TO OBTAIN AN ADDITIVE CONJOINT
SCALING SOLUTION.


TITLE
ACB DESIGN.  4X4X3 DESIGN.  $L, PL, PW.
PRINT=INPUT,ITERATIONS=25
TIES SECONDARY
*  EXAMPLE NO. 1.
*
*  SAMPLE DATA, SUBJECT 1, ATTRACTIVENESS.
DATA
   3   4   4   3   1   1   3   1   0
(3F6.0)
    58    67    68
    77    78    74
    86    94    91
    56    57    64
    74    75    77
    85    89    91
    54    55    59
    64    71    73
    51    86    88
    79    89    89
    91    93    94
    48    69    67
    56    62    69
    68    73    72
    76    87    86
    81    49    49
    52    59    65
    76    78    77
    38    41    32
    52    52    53
    59    64    69
    77    79    83
    88    91    93
    59    59    61
    51    49    48
    55    64    65
    64    72    76
```

```
    51    35    36
    49    49    48
    57    63    59
    42    21    21
    36    34    36
    79    49    48
    59    78    77
    85    88    88
    55    54    57
    39    41    41
    61    54    54
    61    67    69
    35    22    23
    36    38    35
    52    47    48
     9     9     8
    21    25    19
    39    32    29
    69    72    71
    52    86    87
    58    76    75
COMPUTE
TITLE
ACB DESIGN.  4X4X3 DESIGN.  $L, PL, PW.
PRINT=INPUT,ITERATIONS=25
TIES SECONDARY
*  EXAMPLE NO. 2.
*
*  SAMPLE DATA, SUBJECT 1, RISKINESS.
DATA
   3   4   4   3   1   1   3   1   0
(3F6.0)
     3     5     5
     4     5     5
     4     5     6
     9     9     9
     8     9     9
     9     9     9
    14    15    15
    15    15    15
    15    15    15
    19    19    19
    19    19    19
    19    19    19
     9     9     9
     9     9     9
     6     9     9
    19    19    19
    18    19    19
```

```
     19     19     19
     52     52     51
     29     29     29
     29     28     29
     89     89     89
     38     39     39
     39     39     39
     15     15     16
     15     14     15
     14     14     15
     51     52     51
     29     29     29
     29     29     29
     92     91     91
     61     51     52
     46     45     45
     96     96     92
     95     94     94
     59     59     59
     19     19     19
     19     19     19
     19     19     19
     88     89     89
     39     39     89
     39     39     39
     94     96     95
     96     93     94
     59     59     59
     98     99     96
     97     98     96
     98     97     96
COMPUTE
TITLE
KRUSKAL'S SQUARE, MONANOVA EXAMPLE.
PRINT=INPUT,ITERATIONS=25
TIES SECONDARY
*  EXAMPLE NO. 3.
*
*  SAMPLE DATA, SQUARED SCALE VALUES
DATA
   2   3   3   1   1   1   1   1   0
(3F4.0)
   1   4   9
   4   9  16
   9  16  25
COMPUTE
STOP
```

N O N M E T R G

```
*****************************************************
*                                                   *
*  N  O  N  M  E  T  R  G                           *
*                                                   *
*  INSTRUCTIONS FOR USING NONMETRG                  *
*                                                   *
*****************************************************
*  NONMETRG IS A CONJOINT ANALYSIS PROGRAM          *
*  DESIGNED TO USE A "NONMETRIC REGRESSION"         *
*  PROCEDURE FOR OBTAINING AN ADDITIVE CONJOINT     *
*  SCALING OF A SET OF MULTI-ATTRIBUTE STIMULI.     *
*  THE PROGRAM WAS ORIGINALLY WRITTEN BY            *
*  RICHARD M. JOHNSON AND WAS MODIFIED,             *
*  EXTENDED, AND ADAPTED FOR USE ON THE AMDAHL      *
*  470 SYSTEM AT THE OHIO STATE UNIVERSITY BY:      *
*  DR. THOMAS E. NYGREN                             *
*  DEPT. OF PSYCHOLOGY                              *
*  OHIO STATE UNIVERSITY                            *
*                                                   *
*****************************************************

*****************************************************************
*                                                               *
*  TO CALL THIS PROGRAM FOR THE SOURCE DECK FORM FROM           *
*  THE TAPE "CPSCAL", USE THE FOLLOWING JCL CARDS:              *
*                                                               *
*   1)  YOUR ID CARD                                            *
*   2)  //  TIME=2,REGION=550K                                  *
*   3)  /*JOBPARM LINES=10000,DISKIO=200,TAPEIO=200           *
*   4)  /*SETUP UNIT=TAPE9,ID=(CPSCAL,XXXX,READ)             *
*   5)  //  EXEC FORTHCLG,TIME.GO=2,REGION.GO=550K           *
*   6)  //FORT.SYSIN DD UNIT=3400,VOL=SER=CPSCAL,            *
*   7)  //  DISP=(OLD,PASS),DSN=NONMETRG.VER2,               *
*   8)  //  LABEL=(6,SL),                                    *
*   9)  //  DCB=(RECFM=FB,LRECL=80,BLKSIZE=1600)             *
*  10)  //GO.SYSIN DD *                                      *
*                                                               *
*  11)  INPUT DECK AS DESCRIBED BELOW (CONTROL AND              *
*       DATA CARDS GO HERE.)                                    *
*                                                               *
*  12)  /*                                                     *
*  13)  //                                                     *
*                                                               *
*****************************************************************
```

****
* INPUT DECK ARRANGEMENT *
****

CARD(S) A. (JCL CARDS- SEE ABOVE CARDS 1 THROUGH 10.)

CARD B. PARAMETER CARD. (FORMAT IS 14I4.)

COL.	PARAMETER	MEANING
1- 4	NF	NUMBER OF FACTORS. MAX=5.
5- 8	M	TOTAL NUMBER OF LEVELS OF ALL FACTORS. MAX=36 FOR VER2. MAX=24 FOR VER1.
9-12	NBLKS	NUMBER OF BLOCKS OR TRADE-OFF MATRICES PRESENT IN THE DATA. MAX=10 FOR VER2. MAX=6 FOR VER1. THE MAXIMUM NUMBER OF STIMULI IN EACH BLOCK IS 260 FOR VER2 AND IS 150 FOR VER1.
13-16	ITRLIM	NUMBER OF ITERATIONS ALLOWED TO REACH OPTIMAL SCALING CRITERION. (THE' DEFAULT VALUE IS 30; MAX=50.)
17-20	ICOT	=0, NO WEIGHTING IS DESIRED. =1, WEIGHTING IS DESIRED.
21-24	ITIES	=0, IF TIES IN THE DATA ARE TO BE LEFT AS TIES IN THE SCALING SOLUTION. =1, IF TIES ARE NOT TO BE FORCED IN THE SCALING SOLUTION. NONMETRG WILL BREAK TIES AS NECESSARY TO IMPROVE THE FIT OF THE SCALING SOLUTION.
25-28	LABEL	=0, NO LABELS ARE PROVIDED FOR DESCRIBING LEVELS OF FACTORS. FACTOR LEVELS WILL BE NUMBERED FROM '1' TO 'M'. =1, LABELS ARE PROVIDED BY THE USER.
29-32	NPUN	=0, DO NOT PUNCH SCALING SOLUTION ON CARDS. =1, SCALING SOLUTION WILL BE PUNCHED
33-36	NREP	NUMBER OF DATA MATRICES(SUBJECTS, REPLICATIONS) TO BE INDEPENDENTLY RESCALED.
37-40	LASTIT	=0, USE THE SOLUTION FOR THE ITERATION WITH THE LOWEST 'THETA' VALUE. =1, USE THE SOLUTION FROM THE LAST ITERATION.

41-44	NREVR	=0 OR <0, THE INPUT DATA WILL BE GIVEN VALUES OF THE OPPOSITE SIGN. THAT IS, THE DATA WILL BE REVERSED. =1 OR >1, THE INPUT DATA WILL BE LEFT AS IS.
45-48	IPLOT	=0, NO PLOTS WILL BE DRAWN. =1, A PLOT OF THE ORIGINAL DATA (X-AXIS) VS. THE RESCALED ADDITIVE STIMULUS VALUES (Y-AXIS) WILL BE DRAWN FOR EACH BLOCK (I.E., 'NBLKS').
49-52	JUNIT	=0, INPUT DATA VALUES ARE ON PUNCHED CARDS. =N, INPUT DATA VALUES ARE ON LOGICAL UNIT NUMBER 'N'.
53-56	NCARD	NUMBER OF TITLE OR DESCRIPTION CARDS USED. (NO LIMIT.)

CARD C. CRITERION AND START CARD. (FORMAT IS 2F12.6).

COL.	PARAMETER	MEANING
1-12	CRITR	IMPROVEMENT CRITERION VALUE FOR STOPPING THE ITERATIVE PROCEDURE. (A TYPICAL VALUE IS 0.0001.)
13-24	START	A REAL-VALUED ADDITIVE CONSTANT TO BE ADDED TO THE SCALE VALUE OF EACH STIMULUS IN THE ANALYSIS. THIS VALUE IS USUALLY LEFT AS 0.0.

CARD(S) D. LABELS CARD(S). (OPTIONAL).

THESE CARDS ARE OPTIONAL AND WILL BE INCLUDED ONLY IF THE "LABEL" PARAMETER ON CARD B, COLUMNS 25-28, IS "1". FORMAT IS (10A8). EACH CARD CONTAINS, IN ORDER, THE LABELS OF THE FACTOR LEVELS. THERE CAN BE A MAXIMUM OF TEN LABELS PER CARD. EACH LABEL CAN BE UP TO EIGHT CHARACTERS IN LENGTH. USE AS MANY LABEL CARDS AS NEEDED.

COL.	MEANING
1- 8	LABEL FOR LEVEL 1 OF FACTOR 1
9-16	LABEL FOR LEVEL 2 OF FACTOR 1
. ETC.	
.	
.	

CARD(S) E. TITLE CARD(S). (FORMAT IS 20A4).

USE AS MANY CARDS AS ARE SPECIFIED BY THE PARAMETER 'NCARD' ON CARD B, COLUMNS 53-56.

CARD F. STIMULI (OR DATA CELLS) PER BLOCK CARD (FORMAT IS 16I4.)

COL.	PARAMETER	MEANING
1- 4	NV(1)	NUMBER OF STIMULI OR DATA CELLS IN BLOCK 1.
5- 8	NV(2)	NUMBER OF STIMULI OR DATA CELLS IN BLOCK 2.
. ETC.		
.		
.		

CARD G. LEVELS OF FACTOR CARD. (FORMAT IS 16I4).

COL.	PARAMETER	MEANING
1- 4	NV(1)	NUMBER OF LEVELS OF FACTOR 1.
5- 8	NV(2)	NUMBER OF LEVELS OF FACTOR 2.
. ETC.		
.		
.		

CARD H. FORMAT FOR READING IN THE INITIAL RANDOM VECTOR. (FORMAT IS 20A4.)

CARD I. INITIAL RANDOM VECTOR.

CARD I MUST CONTAIN "M" VALUES. THESE VALUES ARE USED TO GENERATE THE INITIAL SCALING OF THE DATA AND TO BEGIN THE ITERATIVE PROCEDURE.
RANDOM START DATA MUST BE REAL-VALUED.

CARD J. FORMAT FOR READING IN THE DATA. (FORMAT IS 20A4.)
DATA MUST BE REAL-VALUED NUMBERS.

CARD K. DATA MATRICES.

ALL 'NREP' DATA MATRICES WILL BE PLACED HERE, ONE BEHIND THE OTHER. FORMAT MUST CONFORM WITH CARD J. EACH CARD CONTAINS INFORMATION FOR ONE DATA POINT. THERE ARE 'M+1' VALUES. THE FIRST M VALUES ARE 1'S AND 0'S, SIGNIFYING THE PRESENCE OR ABSENCE OF EACH OF THE 'M' LEVELS OF THE FACTORS IN DEFINING THE CELL LOCATION OF THE DATA POINT. THE "M+1"ST VALUE IS THE ACTUAL DATA VALUE.

CARD L. END OF ANALYSIS CARD.

A BLANK CARD TO SIGNIFY THE END OF THE ANALYSIS. IF ADDITIONAL ANALYSES ARE INCLUDED, REPEAT CARDS A THROUGH K. THE BLANK CARD IS THE LAST CARD IN THE DECK.

```
   ************************************************
   *                                              *
   *  N O N M E T R G . E X A M P L E             *
   *                                              *
   ************************************************

THE FOLLOWING DATA CARDS ILLUSTRATE THREE DIFFERENT EXAMPLES
USING THE NONMETRG PROGRAM.  EXAMPLE 1 CONSISTS OF AN
ANALYSIS OF A 4X3X4 COMPLETE DESIGN.  EXAMPLE 2 CONSISTS
OF A 3X3X3 DESIGN IN WHICH THE DATA WERE OBTAINED FROM
THREE SEPARATE 3X3 TRADE-OFF MATRICES.  EXAMPLE 3 USES
KRUSKAL'S TWO-FACTOR PERFECT SQUARES DATA PRESENTED IN A
3X3 DESIGN.



   3  11   1  30   0   0   1   0   1   1   1   1   0  15
    0.001000    0.000000
 $L=-.10 $L=-.20 $L=-.30 $L=-.40  PW=2/8  PW=3/8  PW=4/8  PL=1/8  PL=2/8
  PL=4/8
EXAMPLE NO. 1.     NONMETRG.

CONJOINT SCALING:  ATTRACTIVENESS DATA.

 1  SUBJECT.
   48  STIMULI.   1X4X3X4 DESIGN.

FACTORS ARE:
AMOUNT TO LOSE, 4 LEVELS.          -10, -20, -30, AND -40 CENTS.
PROBABILITY OF WINNING, 3 LEVELS.  2/8, 3/8, AND 4/8.
PROBABILITY OF LOSING, 4 LEVELS.   1/8, 2/8, 3/8, AND 4/8.

DATA ARE MEAN ATTRACTIVENESS RATINGS FOR THE 48 GAMBLES.
SCALE RANGES FROM '1' TO '100'.
STIMULI ARE IN THE NATURAL ORDER.
  48
   4   3   4
(11F1.0)
35687946655
(11X,11F1.0,F8.3)
    1   1  1000100 1000   81.333
    1   2  1000 1000100   70.000
    1   3  1000 1000010   67.000
    1   4  1000 1000001   67.667
    1   5  1000010 1000   79.667
    1   6  1000010 0100   71.000
    1   7  1000010 0010   72.333
    1   8  1000010 0001   75.000
    1   9  10000C 11000   93.000
```

1	10	100000 10 100	86.333
1	11	100000 10010	81.333
1	12	100000 10001	74.667
1	13	0100 100 1000	60.667
1	14	0100 1000 100	69.000
1	15	0100 1000010	37.000
1	16	0100 1000001	24.333
1	17	0100010 1000	79.333
1	18	0100010010 0	61.333
1	19	0100010001 0	61.333
1	20	0100010000 1	48.000
1	21	0100001 1000	86.333
1	22	0100001 0 100	74.333
1	23	0100001 0010	68.667
1	24	0100001 0001	66.333
1	25	0010 100 1000	70.000
1	26	0010 1000 100	51.333
1	27	0010 1000010	31.000
1	28	0010 1000001	24.333
1	29	0010010 1000	82.667
1	30	0010010010 0	58.667
1	31	0010010001 0	47.000
1	32	0010010000 1	29.667
1	33	0010001 1000	80.667
1	34	0010001 0 100	76.333
1	35	0010001 0010	64.000
1	36	0010001 0001	52.000
1	37	0001100 1000	67.667
1	38	0001100 0100	37.333
1	39	0001100 0010	29.000
1	40	0001100 0001	25.000
1	41	0001010 1000	83.000
1	42	0001010 0100	61.333
1	43	0001010 0010	33.000
1	44	0001010 0001	38.000
1	45	0001001 1000	77.000
1	46	0001001 0100	66.000
1	47	0001001 0010	57.667
1	48	0001001 0001	32.333

```
   3   9   3  20   0   0   1   0   1   1   1   1   0  10
    0.001000    0.000000
GAS15-24GAS25-34GAS35-44 LOW-DEP MED-DEP  HI-DEPSTYLE-PUSTYLE-AVSTYLE-MD
EXAMPLE NO. 2.     NONMETRG.

CONJOINT EXAMPLE: CAR ATTRIBUTE UTILITIES.   (RANKS)

3 FACTORS.
3 LEVELS OF EACH FACTOR.
3 BLOCKS.  EACH BLOCK  IS A 3X3 RANKING OF 9 COMBINATIONS.
9 TOTAL ATTRIBUTE LEVELS TO SCALE.

   DATA ARE RANKS.
   9   9   9
   3   3   3
(9F1.0)
983265741
(9F1.0,F7.0)
100100000     9.0
010100000     8.0
001100000     6.0
100010000     7.0
010010000     4.0
001010000     3.0
100001000     5.0
010001000     2.0
001001000     1.0
100000100     9.0
010000100     6.0
001000100     3.0
100000010     8.0
010000010     5.0
001000010     2.0
100000001     7.0
010000001     4.0
001000001     1.0
000100100     9.0
000010100     6.0
000001100     3.0
000100010     8.0
000010010     5.0
000001010     2.0
000100001     7.0
000010001     4.0
000001001     1.0
```

```
   2   6   1  30   0   0   0   0   1   1   1   1   0   4
    0.000100     0.000000
EXAMPLE NO. 3.      NONMETRG.

KRUSKAL'S MONANOVA EXAMPLE.
DATA ARE SQUARES IN A  3 X 3 DESIGN.
   9
   3   3
(6P4.0)
   3   8   9   4   1   7
(6F1.0,F6.0)
100100      1
100010      4
100001      9
010100      4
010010      9
010001     16
001100      9
001010     16
001001     25
```

P C J M 2

```
*****************************************************
*                                                   *
*  P   C   J   M   2                                *
*                                                   *
*  INSTRUCTIONS FOR USING PCJM2                     *
*                                                   *
*****************************************************
*****************************************************
*                                                   *
*  THE PROGRAM HAS BEEN MODIFIED AND ADAPTED FOR    *
*  USE ON THE AMDAHL 470 OPERATING SYSTEM AT        *
*  THE OHIO STATE UNIVERSITY BY:                    *
*  DR. THOMAS E. NYGREN                             *
*  DEPT. OF PSYCHOLOGY                              *
*  OHIO STATE UNIVERSITY                            *
*                                                   *
*****************************************************

*************************************************************
*                                                           *
*  TO CALL THIS PROGRAM FOR THE SOURCE DECK FORM FROM       *
*  THE TAPE "CPSCAL", USE THE FOLLOWING JCL CARDS:          *
*                                                           *
*  1)  YOUR ID CARD                                         *
*  2)  //  TIME=1,REGION=300K                               *
*  3)  /*JOBPARM LINES=10000,DISKIO=500,TAPEIO=500         *
*  4)  /*SETUP UNIT=TAPE9,ID=(CPSCAL,XXXX,READ)           *
*  5)  //  EXEC FORTHCLG,TIME.GO=1,REGION.GO=300K          *
*  6)  //FORT.SYSIN DD UNIT=3400,VOL=SER=CPSCAL,          *
*  7)  //  DISP=(OLD,PASS),DSN=PCJM,                       *
*  8)  //  LABEL=(7,SL),                                   *
*  9)  //  DCB=(RECFM=FB,LRECL=80,BLKSIZE=1600)           *
*                                                           *
* 10)  //GO.SYSIN DD *                                     *
*                                                           *
* 11)  INPUT DECK AS DESCRIBED BELOW (CONTROL AND           *
*      DATA CARDS GO HERE.)                                 *
*                                                           *
* 12)  /*                                                  *
* 13)  //                                                  *
*                                                           *
*************************************************************
```

****

*

* I N P U T D E C K A R R A N G E M E N T *

*

****

CARD(S) A. (JCL CARDS- SEE ABOVE CARDS 1 THROUGH 10.)

CARD B. PARAMETER CARD.

COL.	PARAMETER	MEANING
1- 4	ID	AN IDENTIFICATION NUMBER TO IDENTIFY THE SUBJECT'S DATA SET.
5- 8	DIM1	NUMBER OF LEVELS OF THE FIRST FACTOR. (MAX = 6.)
9-12	DIM2	NUMBER OF LEVELS OF THE SECOND FACTOR. (MAX = 6.)
13-16	DIM3	NUMBER OF LEVELS OF THE THIRD FACTOR. (MAX = 6.)
17-20	PRINT	= 0, IF NONE OF THE VIOLATIONS OF THE AXIOMS ARE TO BE LISTED. THAT IS, THE USER HAS THE OPTION OF HAVING THE PROGRAM LIST ALL OR PART OF THE SET OF VIOLATIONS OF EACH AXIOM. IF "0" IS SPECIFIED, THE VIOLATIONS WILL NOT BE PRINTED. = N, IF "N" VIOLATIONS OF EACH AXIOMS ARE TO BE PRINTED. THE USER IS CAUTIONED TO CHOSE ONLY A MODERATE VALUE OF "N" SINCE AN EXTENSIVE AMOUNT OF PRINTED LINES COULD RESULT.
21-24	SUBCAL(1)	= 0, IF INDEPENDENCE AMONG THE FACTORS IS NOT TO BE TESTED. = 1, IF INDEPENDENCE IS TO BE TESTED FOR THE ORIGINAL FULL MATRIX ONLY. = 2, IF INDEPENDENCE IS TO BE TESTED FOR THE ORIGINAL FULL MATRIX AND ANY SPECIFIED SUBMATRIX.
25-28	SUBCAL(2)	= 0, 1 OR 2 AS FOR SUBCAL(1) ABOVE. SUBCAL(2) IS THE INDICATOR FOR DOUBLE CANCELLATION.
29-32	SUBCAL(3)	= 0, 1 OR 2 AS FOR SUBCAL(1) ABOVE. SUBCAL(3) IS THE INDICATOR FOR JOINT INDEPENDENCE.
33-36	SUBCAL(4)	= 0, 1 OR 2 AS FOR SUBCAL(1) ABOVE. SUBCAL(4) IS THE INDICATOR FOR DISTRIBUTIVE CANCELLATION.

37-40	SUBCAL(5)	= 0, 1 OR 2 AS FOR SUBCAL(1) ABOVE. SUBCAL(5) IS THE INDICATOR FOR DUAL-DISTRIBUTIVE CANCELLATION.
41-44	FLAG	= 0, IF DATA CONSIST OF ONE OBSERVATION PER CELL. = 1, IF DATA CONSIST OF THREE OR MORE OBSERVATIONS PER CELL. TWO OBSERVATIONS PER CELL IS NOT PERMITTED.
45-48	NSUB	= 0, 1, 2, 3, 4 OR 5. NSUB INDICATES THE NUMBER OF SUBMATRICES TO BE ANALYZED. IF NSUB IS SET TO "0", THEN ONLY THE FULL MATRIX WILL BE ANALYZED.
49-52	INTYP	IF EQUAL TO -1, -2 OR -3, IT INDICATES ONE DATA POINT PER CELL. = -1, IF DATA ARE IN A RANDOM OR NON-NATURAL ORDER, ONE OBSERVATION PER CARD. = -2, IF THE DATA ARE IN THE NATURAL ORDER, STRUNG OUT OR ONE PER CARD. = -3, IF DATA ARE IN THE REVERSE NATURAL ORDER, STRUNG OUT OR ONE PER CARD. IN THE REVERSE NATURAL ORDER, FACTOR 1 CHANGES INDICES FASTEST, FOLLOWED BY FACTORS 2 AND 3. IF EQUAL TO 1, 2, 3, 4, 5 OR 6, IT INDICATES MULTIPLE OBSERVATIONS PER CELL. = 1, IF DATA ARE IN A RANDOM OR NON NATURAL ORDER, ONE REPLICATION PER CARD. = 2, IF DATA ARE IN THE NATURAL ORDER WITH ONE REPLICATION PER CARD. = 3, IF DATA ARE IN THE REVERSE NATURAL ORDER WITH ONE REPLICATION PER CARD. = 4, IF DATA ARE IN A RANDOM OR NON NATURAL ORDER, ALL REPLICATIONS PER CARD. = 5, IF DATA ARE IN THE NATURAL ORDER WITH ALL REPLICATIONS ON A CARD. = 6, IF DATA ARE IN THE REVERSE NATURAL ORDER WITH ALL REPLICATIONS ON A CARD. IF EQUAL TO 1 OR 4, THE NUMBER OF REPLICATIONS NEED NOT BE THE SAME IN EACH CELL. IF EQUAL TO 2, 3, 5 OR 6, THE NUMBER OF REPLICATIONS IS ASSUMED TO BE EQUAL FOR ALL CELLS.

53-56	JUNIT	= 0, IF INPUT DATA ARE ON PUNCHED CARDS. = N, IF INPUT DATA ARE ON LOGICAL UNIT NUMBER "N".
57-60	NCARD	NUMBER OF TITLE OR DESCRIPTION CARDS USED. (NO LIMIT.)

CARD C. OUTSIDE FACTOR CARD. (FORMAT IS 614.)

THESE CARDS APPLY TO TESTS OF DISTRIBUTIVE AND DUAL - DISTRIBUTIVE CANCELLATION. FOR BOTH OF THESE AXIOMS, ONE FACTOR IS CONSIDERED THE "OUTSIDE" FACTOR. FOR EXAMPLE, FOR DISTRIBUTIVE CANCELLATION WE COULD HAVE THE MODELS:

(A + B) C, WHERE C IS THE OUTSIDE FACTOR;
(A + C) B, WHERE B IS THE OUTSIDE FACTOR;
(B + C) A, WHERE A IS THE OUTSIDE FACTOR.

COL.	PARAMETER	MEANING
1- 4	INDIC1(1)	= 1, IF DISTRIBUTIVE CANCELLATION WITH FACTOR A AS THE OUTSIDE FACTOR IS TO BE TESTED. = 0, IF TEST IS NOT TO BE DONE.
5- 8	INDIC1(2)	= 1, IF DISTRIBUTIVE CANCELLATION WITH FACTOR B AS THE OUTSIDE FACTOR IS TO BE TESTED. = 0, IF TEST IS NOT TO BE DONE.
9-12	INDIC1(3)	= 1, IF DISTRIBUTIVE CANCELLATION WITH FACTOR C AS THE OUTSIDE FACTOR IS TO BE TESTED. = 0, IF TEST IS NOT TO BE DONE.
13-16	INDIC2(1)	= 1, IF DUAL-DISTRIBUTIVE CANCELLATION WITH FACTOR A AS THE OUTSIDE FACTOR IS TO BE TESTED. = 0, IF TEST IS NOT TO BE DONE.
17-20	INDIC2(2)	= 1, IF DUAL-DISTRIBUTIVE CANCELLATION WITH FACTOR B AS THE OUTSIDE FACTOR IS TO BE TESTED. = 0, IF TEST IS NOT TO BE DONE.
21-24	INDIC2(3)	= 1, IF DUAL-DISTRIBUTIVE CANCELLATION WITH FACTOR C AS THE OUTSIDE FACTOR IS TO BE TESTED. = 0, IF TEST IS NOT TO BE DONE.

CARD D. MISSING DATA INDICATOR. (FORMAT IS F4.0.)

COL.	PARAMETER	MEANING
1- 4	EMPTY	A REAL-VALUED NUMBER INDICATIN NG THE CUTOFF FOR DATA TO BE TREATED AS MISSING. ALL OBSERVATIONS EQUAL TO OR LESS THAN "EMPTY" WILL BE IGNORED.

CARD E. FORMAT FOR READING IN THE DATA. (FORMAT IS 20A4.)

CARD(S) F. TITLE CARDS.
USE AS MANY CARDS AS SPECIFIED BY THE PARAMETER "NCARD."

CARD(S) G. SUBMATRICES CARDS. (FORMAT IS 6I4.)

THESE CARDS ARE PRESENT ONLY IF THE PARAMETER "NSUB" IS GREATER THAN ZERO. THERE WILL BE AS MANY CARDS AS INDICATED BY THE "NSUB" PARAMETER. THE CARDS INDICATE FOR EACH SUBMATRIX TO BE TESTED THE STARTING AND ENDING LEVELS OF FACTORS A, B, AND C.

COL.	PARAMETER	MEANING
1- 4	STRTA(I)	INDICATES THE STARTING LEVEL OF FACTOR A FOR SUBMATRIX I.
5- 8	ENDA(I)	INDICATES THE ENDING LEVEL OF FACTOR A FOR SUBMATRIX I.
9-12	STRTP(I)	INDICATES THE STARTING LEVEL OF FACTOR B FOR SUBMATRIX I.
13-16	ENDP(I)	INDICATES THE ENDING LEVEL OF FACTOR B FOR SUBMATRIX I.
17-20	STRTU(I)	INDICATES THE STARTING LEVEL OF FACTOR C FOR SUBMATRIX I.
21-24	ENDU(I)	INDICATES THE ENDING LEVEL OF FACTOR C FOR SUBMATRIX I.

CARD(S) H. DATA CARDS.

THESE CARDS WILL BE DIFFERENT DEPENDING ON THE VALUE OF THE PARAMETER "ITYPE".

IF ITYPE = -1, THEN PROCEED AS FOLLOWS:

FOR EACH DATA CARD THERE SHOULD BE FOUR NUMBERS ON THE CARD PUNCHED IN THE FORMAT SPECIFIED ABOVE.

A	THE LEVEL OF FACTOR A.
B	THE LEVEL OF FACTOR B.
C	THE LEVEL OF FACTOR C.
ENTRY	THE ACTUAL DATA VALUE. A, B, C, AND ENTRY ARE ASSUMED TO BE INTEGERS.

IF ITYPE = -2, THEN PROCEED AS FOLLOWS:

DATA SHOULD BE STRUNG OUT IN NATURAL ORDER. DATA CAN BE ONE OBSERVATION PER CARD OR MULTI OBSERVATIONS PER CARD.

IF ITYPE = -3, THEN PROCEED AS FOLLOWS:

DATA SHOULD BE STRUNG OUT IN REVERSE NATURAL ORDER.

IF ITYPE = 1, PROCEED AS WITH ITYPE = -1. THERE SHOULD BE ONE CARD FOR EACH REPLICATION OF EACH OBSERVATION. THE LAST DATA CARD SHOULD HAVE A = 999.

IF ITYPE = 2, THEN PROCEED AS FOLLOWS:

CARD H1. THE DATA CARDS SHOULD BE PRECEDED BY A CARD THAT HAS THE NUMBER OF REPLICATIONS FOR EACH OBSERVATION PUNCHED IN COLUMNS 1 - 4.

THE DATA CARDS SHOULD HAVE ONE REPLICATION OF EACH OBSERVATION PUNCHED ON THEM.

IF ITYPE = 3, PROCEED AS WITH ITYPE = 2. THE DATA ARE ASSUMED TO BE IN THE REVERSE NATURAL ORDER.

IF ITYPE = 4, THEN PROCEED AS FOLLOWS:

EACH DATA CARD SHOULD HAVE THE FOLLOWING ENTRIES PUNCHED ON THEM:

A	LEVEL OF FACTOR A.
B	LEVEL OF FACTOR B.
C	LEVEL OF FACTOR C.
NE	NUMBER OF REPLICATIONS OF THIS OBSERVATION.
ENTR(1) - ENTR(NE)	"NE" ACTUAL DATA VALUES.
ENTR(NE)	A, B, C, NE, AND ENTR ARE ASSUMED TO BE INTEGERS.

IF ITYPE = 5, THEN PROCEED AS WHEN ITYPE = 2, EXCEPT THAT ALL REPLICATIONS OF EACH OBSERVATION ARE ON THE SAME CARD. DATA ARE ASSUMED TO BE IN THE NATURAL ORDER.

IF ITYPE = 6, THEN PROCEED AS WHEN ITYPE = 2, EXCEPT THAT ALL REPLICATIONS OF EACH OBSERVATION ARE ON THE SAME CARD. DATA ARE ASSUMED TO BE IN THE REVERSE NATURAL ORDER.

CARD I. END OF DATA DECK CARD.
A BLANK CARD IS INSERTED TO INDICATE THE END OF ALL ANALYSES. IF ADDITIONAL ANALYSES ARE TO BE DONE BEGIN AGAIN WITH CARD B.

```
    **************************************************
    *                                                *
    *  P C J M 2 . E X A M P L E                     *
    *                                                *
    **************************************************


    1   4   4   3  10   1   1   1   1   1   1   0   5   0   5
    1   1   1   1   1   1
    0
(3F6.0)
EXAMPLE NO. 1.
PCJM ANALYSIS OF ATTRACTIVENESS RATINGS.
THREE REPLICATIONS OF EACH JUDGMENT.
4 X 4 X 3  DESIGN, REARRANGED.   A X C X B.
TEST ALL AXIOMS.
    3
     58    67    68
     77    78    74
     86    94    91
     56    57    64
     74    75    77
     85    89    91
     54    55    59
     64    71    73
     51    86    88
     79    89    89
     91    93    94
     48    69    67
     56    62    69
     68    73    72
     76    87    86
     81    49    49
     52    59    65
     76    78    77
     38    41    32
     52    52    53
     59    64    69
     77    79    83
     88    91    93
     59    59    61
     51    49    48
     55    64    65
     64    72    76
     51    35    36
     49    49    48
     57    63    59
     42    21    21
     36    34    36
```

```
    79     49     48
    59     78     77
    85     88     88
    55     54     57
    39     41     41
    61     54     54
    61     67     69
    35     22     23
    36     38     35
    52     47     48
     9      9      8
    21     25     19
    39     32     29
    69     72     71
    52     86     87
    58     76     75
   1   4   4   3  10   1   1   1   1   1   1   0   5   0   5
   1   1   1   1   1   1
   0
(3P6.0)
EXAMPLE NO. 2.
PCJM ANALYSIS OF RISKINESS RATINGS.
THREE REPLICATIONS OF EACH JUDGMENT.
4 X 4 X 3  DESIGN, REARRANGED.   A X C X B.
TEST ALL AXIOMS.
   3
     3      5      5
     4      5      5
     4      5      6
     9      9      9
     8      9      9
     9      9      9
    14     15     15
    15     15     15
    15     15     15
    19     19     19
    19     19     19
    19     19     19
     9      9      9
     9      9      9
     6      9      9
    19     19     19
    18     19     19
    19     19     19
    52     52     51
    29     29     29
    29     28     29
```

```
    89    89    89
    38    39    39
    39    39    39
    15    15    16
    15    14    15
    14    14    15
    51    52    51
    29    29    29
    29    29    29
    92    91    91
    61    51    52
    46    45    45
    96    96    92
    95    94    94
    59    59    59
    19    19    19
    19    19    19
    19    19    19
    88    89    89
    39    39    89
    39    39    39
    94    96    95
    96    93    94
    59    59    59
    98    99    96
    97    98    96
    98    97    96
   1   4   4   3  10   1   1   1   1   1   0   0  -2   0   5
   1   1   1   1   1   1
   0
(F6.0)
EXAMPLE NO. 3.
ONE REPLICATION OF EACH JUDGMENT.  NATURAL ORDER.
PCJH ANALYSIS OF ATTRACTIVENESS RATINGS.
4 X 4 X 3  DESIGN, REARRANGED.   A X C X B.
TEST ALL AXIOMS.
    58
    77
    86
    56
    74
    85
    54
    64
    51
    79
    91
    48
```

```
    56
    68
    76
    81
    52
    76
    38
    52
    59
    77
    88
    59
    51
    55
    64
    51
    49
    57
    42
    36
    79
    59
    85
    55
    39
    61
    61
    35
    36
    52
     9
    21
    39
    69
    52
    58
   1   4   4   3  10   1   1   1   1   1   0   0  -2   0   5
   1   1   1   1   1   1
  0
(F6.0)
EXAMPLE NO. 4.
PCJH ANALYSIS OF RISKINESS RATINGS.
ONE REPLICATION OF EACH JUDGMENT.  NATURAL ORDER.
4 X 4 X 3  DESIGN, REARRANGED.   A X C X B.
TEST ALL AXIOMS.
```

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